Modeling of fluid flow in pipeline with the leaks due to the surface

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Abstract

A mathematical model of viscous fluid flow in the pipeline with the presence of leaks due to their surface, which is based on a system of Navier–Stokes equations in two-dimensional rectangular region with a special type of boundary conditions has been designed. The geometric configuration of the leakage zone is taken into the account. It is believed that the fluid motion is under the influence of constant length of pressure difference. For the solving of this system, the numerical method of finite differences was developed by which the finite differences scheme is realized – the first step is implicit in longitudinal coordinate, and the second – on the transversal. The study on the stability of the spectral features method, stability conditions are established for the case of flow calculation with specified parameters and for the given type of the pipeline geometry. The calculations were made for a wide class of boundary conditions that established the patterns of distribution of velocities in different configurations and leakage areas in their absence, it was found that conspicuous effects of leaking occuring in the area around the pipe walls. The pattern of deviations from symmetry of the distribution of values of the longitudinal component was establishe, it depends on the distance of the leak, of the speedchange of the zone near the pipe wall, it also depends on the leakage intensity and the estimated parameters of the grid. It was set that the behavior of the flow in the zone near the wall after the longitudinal component derivative turned zero. The results can be used in designing the localization of small oil leaks from different areas of leakage configuration, the indicated technique can also be used in the study of utility and process piping in many different industries. It was found that the developed method adequately describes the phenomenon. Directions for further research are as follows: identification of dependencies different kinds of liquids, piping specifications, configuration leakage areas and exploration of the case for more complex dependencies pressure on the coordinates of the investigated area.

Key Words: *flow velocity, leakage coordinates, mathematical model, Navier–Stokes system, numerical method, stability.*

Introduction

In the study of actual piping systems, one of the most important issues of control of their technical condition is to detect small leaks of products transported, as they cause substantial economic losses and threats to the environment. This problem is solved by experimental methods [1–7]; however, a significant interest lies in the question of the use of mathematical methods for modeling of these processes. These methods make it possible to explore a wide class of phenomena and processes, which are accompanied by small leakage products. In some cases [8, 9], it is possible to obtain analytical solutions to the problem of fluid flow in the regions with different geometric configurations, but in most practical problems, it is necessary to use numerical methods for solving the issues of such kind. This is because the real technical systems have a complex spatial configuration, and often correct formulation of boundary conditions is either impossible or can be made only in approximate form and size of the area in which the researches require the construction of difference schemes with fixed parameters of the significant volume calculations nets. For a long time, this problem could not be resolved due to the limited capabilities of the computer systems. Modern computing capabilities allow the implementation of sophisticated mathematical models that use a system of Navier–Stokes equations to solve practical problems for engineering systems [10, 11]. The actual problem of estimating parameters of the flow, the universality of mathematical models, and models of viscous fluid flow in particular allows to apply them not only to study of the type of trunk pipeline systems, which is the main part of the proposed article, but also for diagnosing the origins of technological pipelines for different purposes used in cars, reactors, utilities, etc. In this paper, the model for flow in a pipe leaking product is constructed, which includes a system of equations, boundary conditions and the special conditions for the variation of pressure, and difference scheme for this system is implemented, the computational algorithm is build that examines the stability of different schemes for a given performance liquid that is transported, and the velocity field is calculated by geometric pipe parameters. The results of the calculations, discussion, comments, outcomes, specified areas for further research are presented.

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Simulation of the Viscous Fluid Flow in the Pipe under the Steady Pressure Drop

The system of Navier–Stokes equations in primitive variables. When modeling the flow of a viscous fluid through pipes in the general formulation, the system of Navier–Stokes equations in a cylindrical coordinate system is used [8]:

$$\begin{cases} \frac{\partial \upsilon_r}{\partial t} + \boldsymbol{\upsilon}\nabla\upsilon_r - \frac{\upsilon_\varphi}{r} = \\ \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\Delta\upsilon_r - \frac{\upsilon_r}{r^2} - \frac{2}{r^2}\frac{\partial \upsilon_\varphi}{\partial \varphi}\right); \\ \frac{\partial \upsilon_\varphi}{\partial t} + \boldsymbol{\upsilon}\nabla\upsilon_\varphi + \frac{\upsilon_r\upsilon_\varphi}{r} = \\ \qquad = -\frac{1}{\rho r}\frac{\partial p}{\partial \varphi} + \nu\left(\Delta\upsilon_\varphi - \frac{\upsilon_\varphi}{r^2} + \frac{2}{r^2}\frac{\partial \upsilon_r}{\partial \varphi}\right); \\ \frac{\partial \upsilon_z}{\partial t} + \boldsymbol{\upsilon}\nabla\upsilon_z = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\Delta\upsilon; \\ \frac{1}{\rho}\frac{\partial r\upsilon_r}{\partial r} + \frac{1}{r}\frac{\partial \upsilon_\varphi}{\partial \varphi} + \frac{\partial \upsilon_z}{\partial z} = 0, \end{cases} \tag{1}$$

where $\nabla f = \boldsymbol{i}_r\frac{\partial f}{\partial r} + \boldsymbol{i}_\varphi\frac{1}{r}\frac{\partial f}{\partial \varphi} + \boldsymbol{i}_z\frac{\partial f}{\partial z}$,

$\Delta f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 r}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$ – accordingly is the operator gradient and operator Laplacian in the cylindrical coordinate system. The system (1) can be written in a simpler form by the following assumptions:

from a three-dimensional cylindrical coordinate system (1) a transition to a two-dimensional Cartesian system – one of the coordinates is chosen according to the pipe length, and the other – according to its cut;

the behavior of the flow in a rectangular region in the presence of defects is studied; they are modeled as zones of leakage through the surface of the pipe;

a three-dimensional effects are not taken into the account – similar to Poiseuille flow, and the transient nature of the process is not taken into the account as well.

In this case, the system of Navier–Stokes equations is written in the following form:

$$\begin{cases} u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right); \\ u\frac{\partial \upsilon}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 \upsilon}{\partial x^2} + \frac{\partial^2 \upsilon}{\partial y^2}\right); \\ \frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0. \end{cases} \tag{2}$$

It is believed that the pressure along the length of the pipe varies linearly:

$$p = p_0 - kx, \tag{3}$$

where k is the coefficient of pressure drop. The method of solving the Navier–Stokes equations, which is used in the implementation of the components of the velocity vector is called the method of solution using primitive variables [10].

Boundary conditions. It is believed that the current area – rectangle, generally speaking, is of infinite length – thus, the cylindrical pipe is modeled; x – longitudinal coordinate, y – transverse, in fact, along the pipe diameter:

$$\begin{cases} u\big|_{x=0} = -\frac{ky^2}{4\mu} + \frac{kRy}{2\mu}; \\ u\big|_{y=0} = u\big|_{y=2R} = 0; \\ \upsilon\big|_{x=0} = \upsilon\big|_{y=0} = 0; \\ \upsilon\big|_{y=2R} = \begin{cases} 0, \; x \le x_1, x \ge x_2; \\ V_{leak}, x \in [x_1, x_2], \end{cases} \end{cases} \tag{4}$$

where $[x_1, x_2]$ is the piece of leakage, arbitrary configuration of leaks zones in length and intensity sources is possible; μ is the dynamic viscosity of the product transported; R is the pipe radius; ρ is the product density. It is believed that when $x = 0$, the flow of fluid through the pipe is accurately described by the Poiseuille flow model [8]; $\nu = \mu/\rho$ is the kinematic viscosity; V_{leak} is the rate of fluid leakage through the pipe surface.

Difference Schemes for the Navier–Stokes System of Equations

The solution of (2) with boundary conditions (3) is performed by the method of finite differences, where the size is selected.

Δx – step in longitudinal coordinate,

Δy – step in transverse coordinate.

To replace their original counterparts, the following difference-based dependencies are used [12]:

$$\begin{aligned} &\frac{\partial u}{\partial x} = \frac{u_m^{n+1} - u_m^n}{\Delta x}, \quad \frac{\partial \upsilon}{\partial x} = \frac{\upsilon_m^{n+1} - \upsilon_m^n}{\Delta x}; \\ &\frac{\partial u}{\partial y} = \frac{u_{m+1}^{n+1} - u_{m-1}^{n+1}}{2\Delta y}, \quad \frac{\partial \upsilon}{\partial y} = \frac{\upsilon_{m+1}^{n+1} - \upsilon_{m-1}^{n+1}}{2\Delta y}; \\ &\frac{\partial^2 u}{\partial x^2} = \frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{\Delta x^2}, \quad \frac{\partial^2 \upsilon}{\partial x^2} = \frac{\upsilon_m^{n+1} - 2\upsilon_m^n + \upsilon_m^{n-1}}{\Delta x^2}; \\ &\frac{\partial^2 u}{\partial y^2} = \frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{\Delta y^2}, \quad \frac{\partial^2 \upsilon}{\partial y^2} = \frac{\upsilon_m^{n+1} - 2\upsilon_m^n + \upsilon_m^{n-1}}{\Delta y^2}, \end{aligned} \tag{5}$$

difference analogues are used (4) to the system (2), the following difference equation analogues are received:

$$\begin{aligned} &u_m^n\frac{u_m^{n+1} - u_m^n}{\Delta x} + \upsilon_m^n\frac{u_{m+1}^{n+1} - u_{m-1}^{n+1}}{2\Delta y} = \\ &= -\frac{1}{\rho}k + \nu\left(\frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{\Delta x^2}\right) + \nu\left(\frac{u_{m+1}^{n+1} - 2u_m^{n+1} + u_{m-1}^{n+1}}{\Delta y^2}\right); \\ &u_m^n\frac{\upsilon_m^{n+1} - \upsilon_m^n}{\Delta x} + \upsilon_m^n\frac{\upsilon_{m+1}^{n+1} - \upsilon_{m-1}^{n+1}}{2\Delta y} = \\ &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\upsilon_m^{n+1} - 2\upsilon_m^n + \upsilon_m^{n-1}}{\Delta x^2}\right) + \nu\left(\frac{\upsilon_{m+1}^{n+1} - 2\upsilon_m^{n+1} + \upsilon_{m-1}^{n+1}}{\Delta y^2}\right); \\ &\frac{u_m^{n+1} - u_m^n}{\Delta x} = -\frac{\upsilon_{m+1}^{n+1} - \upsilon_{m-1}^{n+1}}{2\Delta y}. \end{aligned} \tag{6}$$

With the aim of takig into the account all of the system equations after the pressure value is given by (3) and is a known as a function of the longitudinal coordinate, the third equation of system (6) is substituted into the second one, and then it acquires the following form:

$$u_m^n \frac{\upsilon_m^{n+1} - \upsilon_m^n}{\Delta x} - \upsilon_m^n \frac{u_m^{n+1} - u_m^n}{2\Delta x} =$$
$$= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\upsilon_m^{n+1} - 2\upsilon_m^n + \upsilon_m^{n-1}}{\Delta x^2}\right) + \nu\left(\frac{\upsilon_{m+1}^{n+1} - 2\upsilon_m^{n+1} + \upsilon_{m-1}^{n+1}}{\Delta y^2}\right).$$

Realization of Computational Algorithm

Equation of the system (6) is written as a system of equations with a three diagonal matrix, that allows the application of the sweep method for its decision [12]:

$$u_{m-1}^{n+1}\left(-\frac{\upsilon_m^n}{2\Delta y} - \frac{\nu}{\Delta y^2}\right) + u_m^{n+1}\left(\frac{u_m^n}{\Delta x} - \frac{\nu}{\Delta x^2} + \frac{2\nu}{\Delta y^2}\right) +$$
$$+ u_{m+1}^{n+1}\left(\frac{\upsilon_m^{n+1}}{2\Delta y} - \frac{\nu}{\Delta y^2}\right) = -\frac{1}{\rho}k + \frac{\left(u_m^n\right)^2}{\Delta x} + \nu\left(\frac{u_m^{n-1} - 2u_m^n}{\Delta x^2}\right);$$
$$\upsilon_{m-1}^{n+1}\left(-\frac{\nu}{\Delta y^2}\right) + \upsilon_m^{n+1}\left(\frac{u_m^n}{\Delta x} - \frac{\nu}{\Delta x^2} + \frac{2\nu}{\Delta y^2}\right) +$$
$$+ \upsilon_{m+1}^{n+1}\left(-\frac{\nu}{\Delta y^2}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\left(u_m^n\right)\left(\upsilon_m^n\right)}{\Delta x} + \qquad (7)$$
$$+ \nu\left(\frac{\upsilon_m^{n-1} - 2\upsilon_m^n}{\Delta x^2}\right) + \upsilon_m^n\left(\frac{u_m^{n+1} - u_m^n}{\Delta x}\right).$$

The coefficients of the system of equations with a three diagonal matrix are written in the following way:

for component u:

$$d_m^1 = s - \frac{\upsilon_m^n}{2\Delta y} - \frac{\nu}{\Delta y^2}; \quad e_m^1 = \frac{u_m^n}{\Delta x} - \frac{\nu}{\Delta x^2} + \frac{2\nu}{\Delta y^2};$$
$$f_m^1 = \frac{\upsilon_m^n}{2\Delta y} - \frac{\nu}{\Delta y^2}; \qquad (8)$$
$$b_m^1 = -\frac{1}{\rho}k + \frac{(u_m^n)^2}{\Delta x} + \nu\left(\frac{u_m^{n-1} - 2u_m^n}{\Delta x^2}\right);$$

for component υ:

$$d_m^2 = -\frac{\nu}{\Delta y^2}; \quad e_m^2 = \frac{u_m^n}{\Delta x} - \frac{\nu}{\Delta x^2} + \frac{2\nu}{\Delta y^2}; \quad f_m^2 = -\frac{\nu}{\Delta y^2};$$
$$b_m^2 = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\left(u_m^n\right)\left(\upsilon_m^n\right)}{\Delta x} + \nu\left(\frac{\upsilon_m^{n-1} - 2\upsilon_m^n}{\Delta x^2}\right) + \qquad (9)$$
$$+ \upsilon_m^n\left(\frac{u_m^{n+1} - u_m^n}{\Delta x}\right).$$

It is necessary to keep the information about a value and on two layers on a co-ordinate x: $n-1$ and n value of speeds on a layer $n+1$ are in the process of solving the problem. For the record of task, an overhead index is cast aside in more compact form, whereupon we receive:

$$d_m u_{m-1} + e_m u_m + f_m u_{m+1} = b_m \ .$$

Taking into account the standard assumption about linearness of dependence between the components u_{m-1} and u_m we receive:

$$u_{m-1} = k_m u_m + L_m;$$
$$d_m k_m u_m + d_m L_m + e_m u_m + f_m u_{m+1} = b_m;$$
$$u_m = -\frac{f_m}{d_m k_m + e_m} u_{m+1} + \frac{b_m - d_m L_m}{d_m k_m + e_m}; \qquad (10)$$
$$k_{m+1} = -\frac{f_m}{d_m k_m + e_m}; \; L_{m+1} = \frac{b_m - d_m L_m}{d_m k_m + e_m}.$$

For the receipt of the system solving (6) with boundary terms (3) the sweep method was twice realized, thus, the form of record of boundary terms is indicated, which allows to take into account the localization of places of liquidsource. It is allowed to define direct and reverse sweep motion of *u* and *v* on the layer of *n+1*, and after the proper reassignment of values on layers an algorithm repeats oneself. The amount of steps of iterative process is determined by the firmness of iterative procedure and necessity of control of area of pipeline of the set length.

Investigation of the Stability of the Difference Scheme

The investigation of the stability of difference schemes is carried out by the spectral feature of stability [12]. Since the first two equations (6) are almost identical, the stability of investigation is carried out only for the first equation. The peculiarity of the spectral features is that the system of Navier–Stokes equations are non-linear, so it is necessary to use the principle of frozen coefficients, whereby the coefficients of the system (6) must be constant. With the implementation of the method of frozen coefficients, the component u_m^n is written as:

$$u_m^n = \lambda^n e^{i\varphi}. \qquad (11)$$

The condition of stability of difference schemes can be written as:

$$|\lambda| \le 1.$$

For the first equation of (5):

$$u_m^n \frac{u_m^{n+1} - u_m^n}{\Delta x} + \upsilon_m^n \frac{u_{m+1}^{n+1} - u_{m-1}^{n+1}}{2\Delta y} =$$
$$= \nu\left(\frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{\Delta x^2}\right) + \nu\left(\frac{u_{m+1}^{n+1} - 2u_m^{n+1} + u_{m-1}^{n+1}}{\Delta y^2}\right),$$

introducing the notation:

$$\frac{\Delta x}{\Delta y} = p; \; u_m^n \approx U = A; \; \frac{\upsilon_m^n}{2} \approx V = B; \; \frac{\nu}{\Delta x} = C, \quad (12)$$

the studied equation takes the form:

$$A\left(u_m^{n+1} - u_m^n\right) + Bp\left(u_{m+1}^{n+1} - u_{m-1}^{n+1}\right) =$$
$$= C\left(u_m^{n+1} - 2u_m^n + u_m^{n-1}\right) + p^2\left(u_{m+1}^{n+1} - 2u_m^{n+1} + u_{m-1}^{n+1}\right).$$

Introduction of the characteristic values (12) allows the selection of such parameters as computational grid, that provide the resistance of computational scheme. In the view of (11) and Euler's formula:

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

derived by:

$$A\left(\lambda^2-\lambda\right)-2Bp\lambda^2\ i\sin\varphi = \\ = C\left(\lambda^2-2\lambda+1\right)-4p^2\lambda^2\sin^2\frac{\varphi}{2}. \quad (13)$$

The equation (13) is a quadratic one with the complex coefficients. Its roots are determined by the relations:

$$\lambda_1 = \frac{A-2C+\sqrt{A^2-8BCp\ i\sin\varphi+4Cp^2\sin^2\frac{\varphi}{2}}}{2(A-2Bp\ i\sin\varphi-C+4p^2\sin^2\frac{\varphi}{2})}; \quad (14)$$

$$\lambda_2 = \frac{A-2C-\sqrt{A^2-8BCp\ i\sin\varphi+4Cp^2\sin^2\frac{\varphi}{2}}}{2(A-2Bp\ i\sin\varphi-C+4p^2\sin^2\frac{\varphi}{2})}. \quad (15)$$

In general, the test conditions $|\lambda_1| \le 1$, $|\lambda_2| \le 1$ requires finding the values $Re\lambda_1$, $Re\lambda_2$, $Im\lambda_1$, $Im\lambda_2$ and testing conditions:

$$\sqrt{(Re\lambda_1)^2+(Im\lambda_1)^2} \le 1; \\ \sqrt{(Re\lambda_2)^2+(Im\lambda_2)^2} \le 1, \quad (16)$$

requiring complex mathematical calculations. However, in this case, the study of the conditions (16) is carried out taking into account the characteristics of the process variables, namely:

$$B \approx 0, \quad C \approx 10^{-5}, \quad A \approx 1. \quad (17)$$

From the equation (14) and inequality (16) in such assumptions the following inequality implies:

$$(16-\varepsilon)\,p^2+64p^4 \ge 0, \quad (18)$$

and from (15), (16) we receive:

$$-\sqrt{1+\varepsilon p^2} \le 1+8p^2, \quad (19)$$

whereas $\varepsilon \approx 10^{-5}$. It is obvious that these inequalities are performed for all values. For large values of leakage, rate difference scheme requires special study, not only in terms of the stability calculation; it is necessary to check the adequacy of the same model (2)–(3) and its relevance to the physical picture of the process. While solving the current problems of the research for which the following conditions take place (17), (18) and (19), the estimated net parameters are chosen with the requirements of the accuracy of calculations

Numerical Characteristics of the Process

Simulation of the flow in the pipeline with defects through whichthe outflow of fluid is conducted for the following flowparameters, the pipe geometry, properties of liquids and gases, the linear pressure drop along the length of pipe:

the average fluid velocity in the pipeline – 2.8 m/s;

typical small leakage rate – up to 50 cm/s;

dynamic viscosity of the fluid (water) – 0.001 kg/cm;

kinematic viscosity – 0.000001 m^2/s;

characteristics of pressure drop – $K = 0.064 - 0.096$;

step on the longitudinal coordinate – 0.08 m;

step on the longitudinal coordinate – 0.025 m, which corresponds to 1.25 m diameter pipe at 50 control points along the transverse coordinate;

number of steps along the longitudinal coordinate – 70000, allowing the calculation of the velocity field for a pipe length of 5.6 km in increments of 8 cm.

Results, Discussion, Feedback and Perspectives

The method of numerical solution of system (2) with discontinuous boundary conditions (3) is implemented as a set of programs for the PC, the complex calculations are conducted, which revealed the following features of the simulated process and its numerical implementation:

the calculation of the velocity field in the pipeline without defects, the transverse velocity component is absent, and the flow of fluid in the pipe is characterized by one velocity component – the longitudinal one. In Fig. 1, the dependence of the maximum and average flow velocity of the liquid in the tube in the absence of defects is demonstrated.

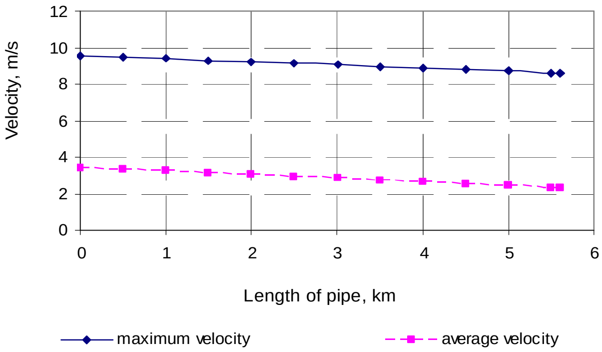


Figure 1 – Maximum and average velocity fluid distribution along the pipeline length without defect

The received numerical solution of the problem differs from the classical result of solving the Poiseuille task, because in solving the problem of Poiseuille flow, it is considered that the velocity profile is independent of the longitudinal coordinate. The obtained numerical results of the system solution (2) with boundary conditions (3) show that the value of the velocity depends on the longitudinal coordinate that corresponds to the real situation of technology – the value of the pressure in the pipeline and, consequently, the magnitude of velocity decreasing the length of the tube; it necessitates pumping and compressor stations at the sites of such type. In Fig. 2, the dependence of the length of the pipe in the parietal area is demonstarted – it changes the coordinate more intensively compared to the maximum and average speeds, the presence of leakage flow pattern changes significantly.

Table 1 shows the results of calculating the longitudinal velocity components in the presence of the defect – leakage of fluid from a pipe at the speed of

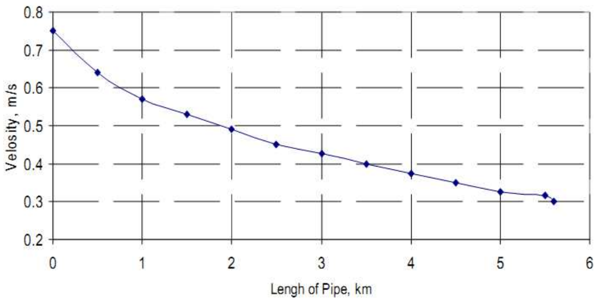


Figure 2 – Velocity fluid distribution on the near-wall area along the pipeline length without defect

Table 1 – The distribution of longitudinal velocity components along the length of pipe in the presence of the defect – source at 10 cm/s at points that were selected in increments of 2.5 sm in diameter

Coordinate on the pipeline diameter, m	Lengh of the pipeline, km						
	0	1	2	3	4	5	5.6
0	0	0	0	0	0	0	0
0.125	3.44	3.24	3.04	2.83	2.63	2.43	2.31
0.250	6.12	5.94	5.76	5.58	5.40	5.21	5.10
0.375	8.03	7.86	7.69	7.51	7.34	7.16	7.06
0.500	9.18	9.01	8.84	8.67	8.49	8.32	8.22
0.625	9.57	9.39	9.22	9.05	8.88	8.71	8.61
0.750	9.18	9.01	8.84	8.67	8.49	8.32	8.22
0.875	8.03	7.86	7.69	7.51	7.34	7.16	7.06
1.000	6.12	5.94	5.76	5.58	5.40	5.21	5.10
1.125	3.44	3.24	3.05	2.84	2.68	2.49	2.38
1.250	0	0	0	0	0	0	0

10 cm/s. It can be concluded that at the speed of 10 m/s, the symmetry, although weakly but is broken, and it applies only to the part of the parietal area defect. When leakage rate is of 1 cm/s, the velocity field (longitudinal component) is almost symmetric, which limits the detection of leaks in relation of using only mathematical methods. When leakage rate is of 40 cm/s, the velocity field (longitudinal component) essentially loses its symmetry – these are the results of the calculation that are presented in Table 2. The parameters of estimated net impact on the simulation results, while increasing their quality painting process have not significantly changed, although quantitative characteristics - value rates – vary by 10–15%, which is acceptable during the technical calculations. The parameters of estimated net – at the diameter of 0.025, 0.08 on longitudinal coordinate, the calculation were adopted as optimal – firstly, they can satisfy the stability conditions (18)–(19), and secondly, with the obtained results correlate well with known theoretical propositions. It should be noted that the implementation of 70,000 steps of iterative procedure design scheme remains stable, and the specified number of iterations is not the limit; in fact, the calculation of the model with no leaks works with further increase in the number of iterations.

Table 2 – The distribution of longitudinal velocity components along the length of pipe in the presence of the defect – source at 40 cm/s at points that were selected in increments of 2.5 sm in diameter

Coordinate on the pipeline diameter, m	Lengh of the pipeline, km						
	0	1	2	3	4	5	5.6
0	0	0	0	0	0	0	0
0.125	3.44	3.24	3.04	2.83	2.63	2.43	2.31
0.250	6.12	5.94	5.76	5.58	5.40	5.21	5.10
0.375	8.03	7.86	7.69	7.51	7.34	7.16	7.06
0.500	9.18	9.01	8.84	8.67	8.49	8.32	8.22
0.625	9.57	9.39	9.22	9.05	8.88	8.71	8.61
0.750	9.18	9.01	8.84	8.67	8.49	8.32	8.22
0.875	8.03	7.86	7.69	7.51	7.34	7.16	7.06
1.000	6.12	5.94	5.76	5.58	5.40	5.21	5.10
1.125	3.44	3.25	3.14	3.11	3.10	3.09	3.08
1.250	0	0	0	0	0	0	0

Fig. 3 demonstrates the results of calculating the maximum deviation from the symmetry of the velocity distribution in the cross section along the length of the pipeline. It is established that noted deviation is maximum, and, in this case the indicated maximum appears at a distance of 1.3 km of the leak.

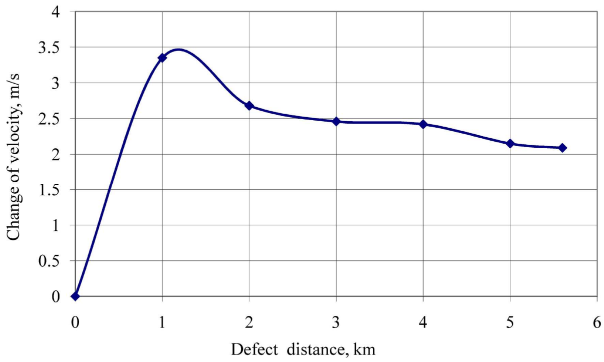


Figure 3 – Deviations from Symmetry Longitudinal Velocity Components Depending on the Distance from the Leak

This can be used in the construction of engineering techniques to identify the coordinates of leaks. In Fig. 4. the results of calculation speed change in parietal area 4 km from the defect, depending on its intensity are demonstrated, it was found that the leak rate of 1 and 10 cm/s, the distribution of indicated velocity is a little different from the case of pipe flow that is without any defects, while at the speeds of 40 and 50 cm/s, the significant (up to 3–5 m/s) velocity deviation at the absence of defects are observed.

Fig. 5 shows the results of calculation of longitudinal velocity fields along at the length of pipe of 1 km for different steps in longitudinal coordinates – 0.08 and 0.05 m, and the pressure distribution pattern is almost identical.

The flow model (2), (3) allows to simulate the flow with the defects located in the opposite sides of the tube to its diameter, and the influence of defects is recorded only in a small parietal area near the leak. The model allows to study the peculiarities of the distribution of velocity in the presence of defects, thus, it is found that the impact of the defect, regardless of the number and coordinates of the points of application alters the current configuration in a small neighborhood of the point source significantly.

Fig. 6 shows the results of simulations of fluid flow in a pipe after the value of the original longitudinal velocity components along the vertical coordinate has become zero (the condition of boundary layer separation [11, 13]).

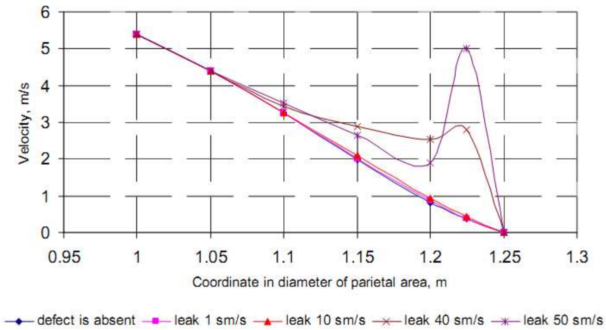
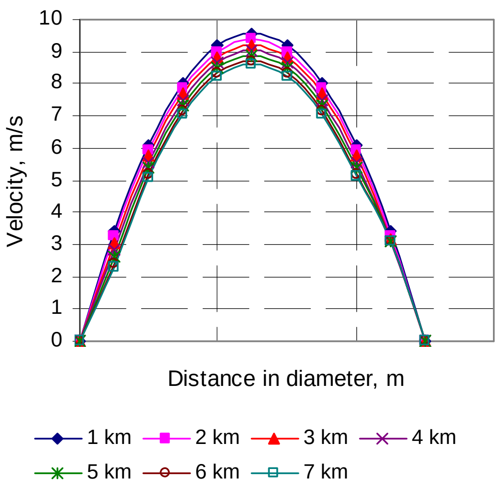
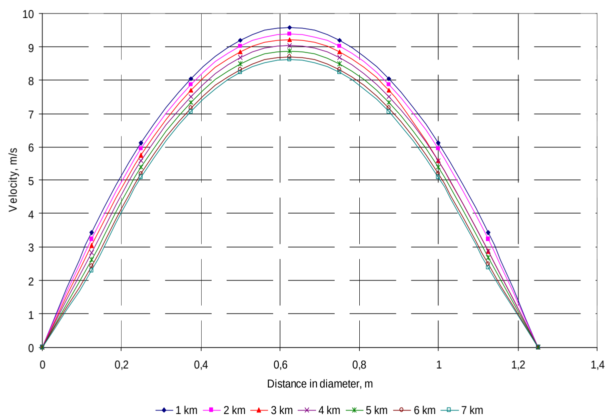


Figure 4 – Speed instability in the near-wall area depending on the leakage rate



a)



b)

Figure 5 – Speed Instability in the Near-Wall Area Depending on the Computational Grid on Longitudinal Coordinate 0,08 m (a) and 0,05 m (b)

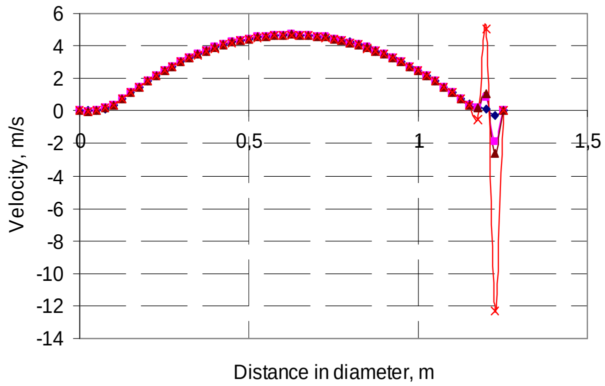


Figure 6 – Speed Instability in the Near-wall Area after the Point of Zero Longitudinal Velocity

In [14], the results of the boundary layer equations for symmetric and axisymmetric flows in the vicinity of the point whereas $\frac{\partial u}{\partial y} = 0$, the methods of constructing solutions of the boundary layer to the point of separation are offered, the cases of continuous flow are indicated, it is proved that the solution of the boundary layer equations may be extended at the point of separation, but the physical meaning, in this case, is lost. In the same paper, the effect of blowing and selection across the border of the body that flowed into the position of separation are explored. These results were confirmed in the calculations that were performed by the model (2)–(3).

The calculation of the specified model stays stable even after passing the point, whereas $\frac{\partial u}{\partial y} = 0$, however, the physical content of the problem is lost: for 2000–3000 steps of iterative procedure, the speed value acquires physically unrealistic values only in a narrow parietal area; in fact, we can say that the section of the pipe narrows through perturbation flow in parietal area, which can cause the occurrence of such negative phenomena as water hammer, thread locking etc. [13]. With the implementation of the model (2)–(3) all of the necessary mathematical formalities are completed [15]: the problem statement and the proof of its correctness, sampling problem and study the stability of the corresponding difference schemes, the choice of a numerical method for solving, programming in a suitable environment and analysis of the obtained numerical results by comparing them with the other experimental and theoretical results. However, [11, 16–18], the basic requirement, which comes to practical solutions of the problems in mechanics is their resistance to various perturbations: in this case, it is the presence of small leaks, which are the points of discontinuity walls and flow transition point, whereas $\frac{\partial u}{\partial y} = 0$.

For the stability analysis, it is important to know the average characteristics, dimensionless parameters and numbers, with the achievement of which its critical values of the transition system are associated in a qualitatively different state, where the contents of the basic equations of the model is lost (for example, lose of the contents of the Navier–Stokes equations). With the implementation of the developed model, the following frequently asked questions raises: whether it is possible to identify the critical parameters using numerical methods in the study of physical process of losing stability, provided that the characteristics of this process are unknown and most are determined by solving certain value problem, which means the loss of stability in numerical simulation is incorrect discrete problem (implementation failure of the following conditions (16), (18), (19)) or a process that occurs in the explored physical body (detachment of the boundary layer, water hammer, the occurrence of turbulent effects)? This significantly concerns the results presented in Fig. 3, 4, 6. At the same time, in this case, these issues can also be interpreted as follows – how to separate the exposure limits in (16), (18), (19) of the physical and mechanical characteristics of the flow parameters and the estimated net? The answer to those questions require further research in the presence of leaks through the surface. Another point, the attention should be paid to during the simulation of fluid flow with the small leaks is the organization of the results calculations output so as to ensure their high information content; at the calculation of velocity fields along the length of the 6 miles pipeline with the increments of 8 cm, it is necessary to detect the patterns of velocity distribution by using the data from the file capacity of 54 MB. Thus, the development of techniques for identifying and eliminating redundant information is needed; it is important in practical implementation techniques.

Conclusions

According to the results of the conducted simulation, the following conclusions can be drawn:

The use of Navier–Stokes equations in the form of (2) with boundary conditions (4) allows to obtain an adequate physical model that can serve as a basis for the construction of diagnostic algorithms of detecting the localization of small leaks in the pipeline. The condition (3) allows, on the one hand, to simplify the calculating algorithm; otherwise, the problem is reduced to the Poiseuille task [10], it is necessary to set an initial approximation of the velocity field and the initial approximation of the pressure field, whereas, in this case, it is necessary to specify only conditions (4);

The used difference schemes tested for stability: the case of flow parameters, which satisfy the condition (17), set the absolute stability of numerical schemes, however, require further research in the issues of sustainability schemes for which the condition (17) is not satisfied;

It is found that the transverse component of velocity varies only in the immediate vicinity of the source area, but it can be concluded that the parameters that define the phenomenon, is the rate of fluid leakage from the pipeline V_{leak}, second flow of fluid Q, and the distance to the leak source L, thus, the further studies should use a diagnostic feature dimensionless parameter $k_1 = \frac{Q_V}{V_{leak} L^2}$ – in the case of volumetric flow, parameter $k_2 = \frac{Q_S}{V_{leak} L}$ – in case of loss through the area; Q_V – flow through value; Q_S – flow through area.

The directions of further research may include: studying of the influence of parameters on the results of difference scheme, the law of pressure change, physical and mechanical properties of liquid transported, and the determination of the relationship between the intensity and magnitude of the leakage area of the steady flow calculation parameters.

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Моделювання течії рідини в трубопроводі за наявності витоків через поверхню

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Побудовано математичну модель течії в'язкої рідини в трубопроводі за наявності витоків через їх поверхню, яка базується на системі рівнянь Нав'є–Стокса в двовимірній прямокутній області зі спеціальним типом граничних умов. Вони враховують геометричну конфігурацію зони витоку. Вважається, що рух рідини здійснюється під дією сталого по довжині перепаду тиску. Для розв'язання вказаної системи розроблено чисельний метод кінцевих різниць, за допомогою якого реалізується схема,яка на першому кроці є неявною по повздовжній координаті, а на другому – по поперечній. Проведено дослідження стійкості за методом спектральної ознаки, встановлено умови стійкості для випадку розрахунку течії із заданими параметрами та для вказаного типу геометрії трубопроводу. Проведено розрахунки для широкого класу граничних умов, встановлено закономірності розподілу швидкостей за різних конфігурацій зон витоку та за їх відсутності, виявлено, що особливо помітними ефекти впливу наявності витоку проявляються в зоні поблизу стінки трубопроводу. Встановлено закономірність відхилення від симетрії розподілу величини повздовжньої компоненти залежно від відстані до місця витоку, зміни швидкості в зоні поблизу стінки труби залежно від інтенсивності витоку та параметрів розрахункової сітки, встановлено особливості поведінки течії в зоні поблизу стінки після набуття повздовжньою компонентою нульового значення. Результати роботи можуть бути використані для розроблення системи локалізації малих витоків нафтопродуктів з різною конфігурацією зон витоків, окрім того, вказана методика може бути використана в дослідженні комунальних трубопроводів, технологічних трубопроводів у різних галузях промисловості. Виявлено, що розроблена методика адекватно описує досліджувані явища. Визначено напрямки подальших досліджень: визначення залежностей для різних типів рідин, характеристик трубопроводів, конфігурації зон витоку, а також дослідження випадку більш складних залежностей тиску від координат досліджуваної області.

Ключові слова: *координати витоку, математична модель, рівняння Нав'є–Стокса, стійкість, чисельний метод, швидкість течії.*