Modeling of ultrasonic guided waves propagation in a waveguide with cross-section of finite size

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Abstract

The most topical task is further development of testing methods of oil and gas industry objects by ultrasonic guided waves. The aim of research is to develop mathematical model of ultrasonic guided waves propagation in oil and gas pipelines made from steel.

The method of research is a computer modelling of ultrasonic guided waves propagation in waveguides with cross-sections of finite size.

Calculations of guided waves propagation have been performed in two spectral ranges. With the increase of frequency an algorithm finds the same number of modes in a more narrow frequency spectra. Numerical results of found modes were estimated by effective mass criterion. The criterion shows that only a few modes from the found set correspond with applied boundary conditions of ultrasonic guided waves propagation.

Results of calculations are applied to ultrasonic guided waves propagation along the V weld. Welded joints sizes meet requirements of normative documents about oil and gas main pipelines.

In the paper it is specified that the parameters of ultrasonic guided waves propagation in a waveguide with its cross-cuts of finite sizes can be calculated utilizing existent algorithms realising search of eigenvalues, based on Timoshenko beam which is a finite element typeIt has been found out that not all results of eigenvalues calculation by the algorithm with boundary conditions that describes propagation of guided waves with specified wavenumber correspond to propagation of the modes. The results of calculations have to be filtered out by the criterion of effective mass. It has been shown that the criterion of effective mass of a mode can be used to determine the type of guided wave. It has been found out that modes, propagating in a waveguide with its cross-cuts of finite sizes, can have marked torsional displacements without additional axial movements.

Key Words: *finite element method, torsional mode, ultrasonic guided wave, ultrasound.*

Introduction

Obtaining of information about parameters of acoustic ways propagation is the main task of acoustic testing. The application of ultrasonic guided waves for the analysis of large industrial objects makes it possible to increase quality and speed of testing significantly. Currently theory of waves propagation is being extensively developed due to use of computers and software which gives the possibility to obtain and analyse large quantities of information about oscillations parameters. All the above mentioned allows for further development of methods aimed at testing of industrial objects of finite cross section size by means of ultrasonic guided waves.

The theory of acoustic guided waves propagation was summarized by [1]. The author singled out the main ways of theory development and introduced the most complete mathematical models describing the connection between the main characteristics of wave propagation in elastic medium with its mechanical parameters and considered the simplest forms of media interfaces. More complete mathematical description of ultrasonic waves interaction with waveguide-ambient medium interface and algorithm of calculation of modes converting have been presented in [2]. Propagation of ultrasonic guided waves in waveguides with heterogeneous properties in the direction of wave propagation has been viewed in [3]. This paper elaborates on a theory of waveguides for acoustic waves. The developed theory of waves propagation as a rule is concluded with complicated transcendental equations or differential second order equations systems. To get solution of equations describing acoustic guided waves propagation it's necessary to develop cumbersome numerical algorithms, the task of which is to get wavefield parameters, using limited resources of personal computers. One of interesting

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approaches towards design of algorithm for guided waves propagation parameters calculation is a combination of an analytical solution and finite elements method [4]. In this paper the authors have used general algorithms of calculations on the basis of parallelepipedal finite element. The solution to acoustic oscillations propagation problem was complicated due to method of boundary conditions assigning and treatment of obtained deformations and tension. A simpler algorithm for calculation of acoustic waves propagation was suggested in [5]. Acoustic oscillations have been described on the basis of beam theory and have made calculations by means of correspondent finite elements.

For the effective use of ultrasonic guided waves in NDT it's necessary to be able to forecast oscillations parameters depending on both mechanical parameters as well as geometrical forms of cross section of the object under investigation. Currently available investigations can sufficiently describe process of oscillations propagation in wavefield, depending on medium mechanical parameters, but such mathematical models are developed for simple geometrical forms of wavefield (plate, cylinder etc.). Real objects have more complicated geometrical forms, without consideration of which it's impossible to determine parameters of ultrasonic guided waves propagation. The aim of investigation is to develop mathematical model of ultrasonic guided waves propagation in wavefield with an arbitrary form of cross section and algorithm of its calculation.

The main mathematical terms, describing ultrasonic guided waves propagation

Ultrasonic waves are described on the basis of basic principles of linear elasticity of preservation of linear angular moments and fundamental equations, describing a relationship between applied force and deformations, based on Newton laws [1]. Mentioned principles are a basis for the law of mechanical energy conservation within the limits of linear elasticity theory. Thus, if force F is applied to one surface of an elastic body, described by unit vector normal line n_j, then on the opposite surface force $F^{(t)}$ will appear (on condition that body isn't moving), which will be equal to:

$$F^{(t)} = \sigma_{ij} n_j \ , \qquad (1)$$

where σ_{ij} is the stress tensor; $i, j = 1, 2, 3$ is the indices, marking Cartesian axes.

The law of angular moments conservation for an elastic isotropic body makes stress tensor symmetrical:

$$\sigma_{ij} = \sigma_{ji} \ . \qquad (2)$$

The law of angular moments conservation can be expressed by means of displacement of elementary volume of elastic medium u_i:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho F_i = \rho \frac{\partial^2 u_i}{\partial t^2} \ , \qquad (3)$$

where ρ is the density; t is the time.

Displacement of elementary volume of elastic medium correlates with deformations ε_{ij} of an elastic body in the following way:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \qquad (4)$$

where x_i is the Cartesian axes.

For elastic medium, described by linear theory of elasticity, stresses and deformations are correlated as follows:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} \ , \qquad (5)$$

where λ, μ is the Lame elasticity constants; δ_{ij} is the Kronecker symbol; $k = 1, 2, 3$.

On the basis of (1)-(5) motion equation of elastic oscillations propagation can be written down:

$$(\lambda + \mu) \frac{\partial^2 u_k}{\partial x_i \partial x_k} + \mu \frac{\partial^2 u_i}{\partial x_j^2} + \rho F_i = \rho \frac{\partial^2 u_i}{\partial t^2} \ . \qquad (6)$$

To calculate elastic oscillations propagation, (5) is better to be presented in a vector form:

$$(\lambda + \mu) \nabla (\nabla \cdot u) + \mu \nabla^2 u + \rho F = \rho \frac{\partial^2 u}{\partial t^2} \ , \qquad (7)$$

where $\mathbf{u} \equiv \{u_1,\ u_2,\ u_3\}$ is the elementary volume displacement vector; $\mathbf{F} \equiv \{F_1,\ F_2,\ F_3\}$ is the force vector;

$$\nabla \equiv \frac{\partial}{\partial x_i} \ ; \ \nabla \cdot \equiv \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_3} \ .$$

Component $\nabla^2 u$ can be represented as follows:

$$\nabla^2 u = \nabla (\nabla \cdot u) - \nabla \times \nabla \times u \ . \qquad (8)$$

Considering (8) equation of elastic oscillations propagation will be:

$$(\lambda + 2\mu) \nabla (\nabla \cdot u) - \mu \nabla \times \nabla \times u + \rho F = \rho \frac{\partial^2 u}{\partial t^2} \ . \qquad (9)$$

where $\nabla \times u_i = \frac{\partial u_i}{\partial x_\alpha} + \frac{\partial u_i}{\partial x_\beta}$; $\alpha, \beta = 1, 2$.

In (9) component $\nabla \cdot u$ describes longitudinal oscillations and $\nabla \times \nabla \times u$ – transverse oscillations. With the help of (1)-(9) any elastic oscillations can be described, but to use equations, describing elastic oscillation propagation it is necessary to use initial and boundary conditions of differential equations. In general immediate use of equations (7) and (9) for engineering objects is not an easy task. It is even more complicated to describe propagation of guided ultrasonic waves by these equations squarely. From the analysis of (9) physical parameters can be singled out which determine form and direction of elastic oscillations propagation (guided waves included):

elastic constants value;

density;

form and value of force applied to elastic body surface;

form of wavefield along which oscillation is propagating.

Helmholtz suggested one of widely used approaches to simplifying of elastic oscillations propagation description [1]. In accordance with his theory, elastic oscillation wavefield can be divided into scalar φ and vector ψ potentials:

$$u = \nabla\varphi + \nabla\times\psi\ , \nabla\cdot\psi = 0\ . \qquad (10)$$

Elastic oscillations propagation with consideration of (9) and (10) and without applied force will be described as follows:

$$(\lambda + 2\mu)\nabla\left(\nabla^2\varphi - \frac{1}{c_l^2}\frac{\partial^2\varphi}{\partial t^2}\right) + \\ + \mu\nabla\times\left(\nabla^2\psi - \frac{1}{c_t^2}\frac{\partial^2\psi}{\partial t^2}\right) = 0\ , \qquad (11)$$

where c_l, c_t is the correspondingly longitudinal and transverse oscillations.

Such oscillations forms are solution to (11):

$$\nabla^2\varphi = \frac{1}{c_l^2}\frac{\partial^2\varphi}{\partial t^2}\ , \nabla^2\psi = \frac{1}{c_t^2}\frac{\partial^2\psi}{\partial t^2}\ . \qquad (12)$$

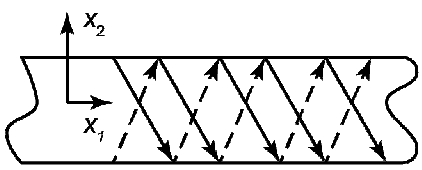
From (12) propagation speeds of longitudinal and transverse waves can be determined:

$$c_l = \sqrt{\frac{\lambda + 2\mu}{\rho}}\ , c_t = \sqrt{\frac{\mu}{\rho}}\ . \qquad (13)$$

Equation (11) describes propagation of elastic oscillations on that part of medium where there is no applied force. But it is supposed that applied force beyond the boundaries of elastic body surface under investigation can be described with the help of harmonic law. Frequency of applied force on the elastic body surface is equal to waves oscillation frequency. Such simplifying assumptions pertain to description of guided waves propagation because they propagate to significant distances from the source of excitation.

From (11)–(13) it can be said that aim of equations solution, describing elastic oscillations propagation is determination of scalar and vector potentials values. If there are potentials values, it is possible to find form of displacement of elementary volume **u** of an elastic body on the whole domain of waves propagation.

The principle of dividing of waves upon oscillations type let us explain guided waves propagation (Fig. 1): longitudinal and transverse oscillations propagating under certain angle towards plate surface create a harmonic oscillation, propagating along its surface in compliance with harmonic law.



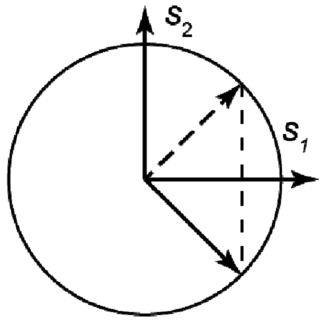
The solid lines indicate incident bulk waves, while the dashed lines indicate reflected bulk waves

Figure 1 – Scheme of guided modes propagation based on partial waves concept

Guided waves occur when there is coincidence of phases of incident and reflected waves from media interface, e.g. elastic body-air (Fig. 2) and creation of standing wave through the thickness of an elastic body (wavefield). Thus, waves within the boundaries of wavefield falling and reflecting from media interface, constructively interference and reconstruct themselves forming wavefield of a stable form. General form for guided wave description will be as follows:

$$u(x,t) = \mathbf{d}\,u(t - \mathbf{s}\,x)\ , \qquad (14)$$

where **d** is the single vector, describing oscillations polarization; **s** is the slowness.



The solid lines indicate incident bulk waves, while the dashed lines indicate reflected bulk waves

Figure 2 – A slowness diagram for guided waves

Polarization of lateral oscillations (Fig. 1) is called vertical polarization and waves are called vertically polarized, but in waveguide horizontally polarized waves Ψ_z can propagate, which are described as follows [1]:

$$\nabla^2\psi_z = \frac{1}{c_t^2}\frac{\partial^2\psi_z}{\partial t^2}\ . \qquad (15)$$

In waveguide of finite cross section size three types of waves will exist as it follows from (12) and (15):

longitudinal wave with polarization of oscillations of elementary volume of elastic medium in the direction of wave propagation;

torsional with polarization of oscillations along waveguide perimeter;

flexural with polarization vertical to axial plane.

Currently theory of guided waves is well-developed for the description of their propagation in round beams. For waveguides of such type setting of boundary conditions by means of applying of cylindrical coordinates system becomes significantly easier. That allowed to obtain analytical equations as a result of direct solution of differential equations, describing propagation of partial waves (bulk waves within the limits of a cylinder). But obtained analytical expressions aren't classical form of equations solution and need development of algorithms of radical numerical search. Analytical solutions become more complicated if we take into account forced oscillations, thus if we specify more complicated boundary conditions the other direction of description of guided ways propagation is the use of materials strength theory within the limits of which longitudinal, torsional and flexural motions in cylindrical beams, bars and plates can be mathematically described. Such motions are described by one equation in partial quartic derivatives. Such a simplified theory actually describes one degree of oscillations freedom and approximates lower part of spectrum in a certain range of a real wavefiled in a waveguide [1]. The advantage of this theory is relatively simple obtaining of a solution. Combination of these two approaches is a theory, suggested by S.P. Timoshenko. According to this theory wave propagation in a beam is described on the basis of materials strength theory but taking into account the whole spectrum of frequencies.

Model for guided waves propagation calculation in a waveguide

To use beams theory by S.P. Timoshenko for waveguides with an arbitrary form of cross section, it's necessary to use finite elements theory. For elastic oscillations propagation tasks theory of semi-analytical method is the best one [4]. According to this theory, analytical solutions are used in direction of elastic wave propagation and finite element approximation of deformations and stresses are used in cross section of waveguide. This method significantly simplifies elastic body points in Cartesian coordinates, making calculations more effective. Finite element nodes in x_1–x_2 can be projected in coordinates ξ_1–ξ_2 in domain $-1 \le \xi_1, \xi_2 \ge 1$, so [7]:

$$\begin{Bmatrix} x_1(\xi_1,\xi_2) = N^T(\xi_1,\xi_2)x^{(1)} \\ x_2(\xi_1,\xi_2) = N^T(\xi_1,\xi_2)x^{(2)} \end{Bmatrix}, \qquad (16)$$

where $x^{(1)} = \left[x_1^{(1)}, x_1^{(2)}, ..., x_1^{(L)}\right]^T$, $x^{(2)} = \left[x_2^{(1)}, x_2^{(2)}, ..., x_2^{(L)}\right]^T$ is the set of finite elements nodes coordinates; L is the number of finite elements in the plane or number of integration points in one finite element; $N(\xi_1,\xi_2) = [N_1, N_2, ..., N_L]^T$ is the transformation matrix.

For three finite elements equally-spaced from each other, elements of transformation matrix are calculated as follows:

$$N_1 = \frac{1}{4}\xi_1(\xi_1 - 1)\xi_2(\xi_2 - 1),$$
$$N_2 = \frac{1}{4}\xi_1(\xi_1 + 1)\xi_2(\xi_2 - 1), \qquad (17)$$
$$N_3 = \frac{1}{4}\xi_1(\xi_1 + 1)\xi_2(\xi_2 + 1).$$

In case of wave propagating, oscillations are supposed to be harmonic in the direction x_3. According to analytical part of a method, displacement is a function of nodes displacement in direction x_3 and time t. Displacements in plane ξ_1–ξ_2 is determined by square law approximation equation. Full displacement of elementary volume of elastic environment $\boldsymbol{\psi}$ is as follows:

$$\psi(\xi_1,\xi_2,\xi_3,t) = \begin{bmatrix} u_1(\xi_1,\xi_2,\xi_3,t) \\ u_2(\xi_1,\xi_2,\xi_3,t) \\ u_3(\xi_1,\xi_2,\xi_3,t) \end{bmatrix} = N(\xi_1,\xi_2)u(\xi_3,t) =$$
$$= \begin{bmatrix} N_1 & 0 & 0 \\ 0 & N_2 & 0 \\ 0 & 0 & N_3 \end{bmatrix} \times \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}. \qquad (18)$$

Equation (18) is a connection between coordinates from domain ξ_1, ξ_2 and real physical processes of elastic environment change.

Elementary volume displacement (18) will describe elastic body deformations through partial displacement derivatives in the following way:

$$\varepsilon(x_1,x_2,x_3,t) = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right) \\ \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}\right) \\ \frac{\partial u_3}{\partial x_3} & \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\right) \end{bmatrix}. \qquad (19)$$

Vital distinction of (19) from deformation determining (4) is its dependence on time. Deformation is presented as a vector, which makes matrix multiplication easier. Transition to partial derivatives in coordinates of domain ξ_1, ξ_2 is realized with the help of Jacobi matrix **J** of tensor analysis theory, which in this case will be:

$$J = \begin{bmatrix} \frac{\partial x_1}{\partial \xi_1} & \frac{\partial x_2}{\partial \xi_1} \\ \frac{\partial x_1}{\partial \xi_2} & \frac{\partial x_2}{\partial \xi_2} \end{bmatrix} =$$
$$= \begin{bmatrix} \frac{\partial N^T(\xi_1,\xi_2)}{\partial \xi_1} x_1^{(L)} & \frac{\partial N^T(\xi_1,\xi_2)}{\partial \xi_1} x_2^{(L)} \\ \frac{\partial N^T(\xi_1,\xi_2)}{\partial \xi_2} x_1^{(L)} & \frac{\partial N^T(\xi_1,\xi_2)}{\partial \xi_2} x_2^{(L)} \end{bmatrix}. \qquad (20)$$

On the basis of (18)–(20) one can build motion equation of elastic medium elementary volume (9), in terms of finite elements algorithm this volume will be equal to one finite element or integration point. To study elastic oscillations propagation across the whole object it's necessary to combine stiffness matrix, mass matrix and force matrix of all elements in accordance with their reciprocal placement [4]:

$$K_1 \frac{\partial^2 u}{\partial \xi_3^2} + K_2 \frac{\partial u}{\partial \xi_3} + K_3 u + M \frac{\partial^2 u}{\partial t^2} = F,$$
$$K_1 = \int_{-1}^{1}\int_{-1}^{1} B_1^H \mathbf{C}\mathbf{B_1} |J| d\xi_1 d\xi_2,$$
$$K_2 = \int_{-1}^{1}\int_{-1}^{1} \left(B_2^H \mathbf{C}\mathbf{B_1} - B_1^H \mathbf{C}\mathbf{B_2}\right)|J| d\xi_1 d\xi_2, \qquad (21)$$
$$K_3 = \int_{-1}^{1}\int_{-1}^{1} B_2^H \mathbf{C}\mathbf{B_2} |J| d\xi_1 d\xi_2,$$
$$M = \int_{-1}^{1}\int_{-1}^{1} \rho N^H N |J| d\xi_1 d\xi_2,$$

where $\mathbf{B_1} = \lfloor B_{\mathbf{1,1}}, B_{1,2}, ..., B_{1,L} \rfloor$ is the coordinates of finite elements nodes; $\mathbf{B_2} = \lfloor B_{\mathbf{2,1}}, B_{2,2}, ..., B_{2,L} \rfloor$ is the differentials of finite elements nodes; **C** is the stiffness matrix of material with dimensions of 6×6; $|J|$ is the determinant of Jacobi matrix; symbol H is the transposed matrix in which each element is conjugated with initial matrix. In case when matrix elements are real but not complex numbers, symbol H will designate only transposed matrix;

$\mathbf{B}_{1,i} = \begin{bmatrix} 0 & 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & 0 & N_i \end{bmatrix}^T$ is the elements of matrix $\mathbf{B}_1$.

Matrix $\mathbf{B}_{2,i}$ elements will be as follows:

$$\begin{bmatrix} \Xi_i J_{11}^{-1} + \Omega_i J_{12}^{-1} & 0 & 0 \\ 0 & \Xi_i J_{22}^{-1} + \Omega_i J_{21}^{-1} & 0 \\ 0 & 0 & 0 \\ \Xi_i J_{21}^{-1} + \Omega_i J_{22}^{-1} & \Xi_i J_{11}^{-1} + \Omega_i J_{12}^{-1} & 0 \\ 0 & 0 & \Xi_i J_{21}^{-1} + \Omega_i J_{22}^{-1} \\ 0 & 0 & \Xi_i J_{11}^{-1} + \Omega_i J_{12}^{-1} \end{bmatrix}, \quad (22)$$

where $\Xi_i = \partial N_i / \partial \xi_1$, $\Omega_i = \partial N_i / \partial \xi_2$.

In (21) integrals should be calculated by Gauss–Legendre algorithm [7]. The advantage is (21) that with the help of this algorithm one can calculate wave propagation at any form of excitation by outer force **F.** In NDT harmonic excitation is most widely used, which in its turn means that bulk ultrasonic waves (longitudinal and transverse), created in elastic medium will be also oscillating in accordance with harmonic law. According to (14) equation, describing guided ultrasonic wave propagation, will be as follows:

$$u(\xi_3, t) = U\, e^{i(k \cdot \xi_3 - \omega t)}, \quad (22)$$

where k is the wave number; ω is the angle frequency; U is the oscillation amplitude.

Oscillation amplitude U for guided wave can be calculated this way [8]:

$$U_m(\xi_3, t) = \upsilon_m\, P_m\, A_m\,, \quad (23)$$

where m is the mode number; υ_m is the coefficient of mode normalizing along waveguide length; P_m is the harmonic propagation coefficient; A_m is the amplitude vector.

Harmonic propagation coefficient P_m is as such [8]:

$$P_m = \begin{bmatrix} \sin(m\pi\pi_3 / L_e) \\ \sin(m\pi\pi_3 / L_e) \\ \cos(m\pi\pi_3 / L_e) \end{bmatrix}^T, \quad (24)$$

where L_e is the waveguide length.

For differential equation of second order (21) and solution form (23) if we take a part of waveguide without exciting force, solution can be found by characteristic values task solution [4]:

$$\left(\begin{pmatrix} 0 & K_3 - \omega^2 M \\ K_3 - \omega^2 M & iK_2 \end{pmatrix} - k \begin{pmatrix} K_3 - \omega^2 M & 0 \\ 0 & K_1 \end{pmatrix} \right) \times \begin{pmatrix} U \\ kU \end{pmatrix} = 0\,. \quad (25)$$

Equation (26) is dispersed as to wave number and angular frequency. As a result of solution (26) form of oscillations can be found. To solve (26) existing algorithms of eigenvalues search can be used, embedded in finite elements calculation software packages.

Calculations and results

Calculating by finite elements method can be carried out according to such algorithm:

1. Waveguide length is set.
2. Cross section form is built.
3. Type of elastic medium is set.
4. Mathematical model describing stress-deformation is selected.
5. Algorithm of mathematical model calculation is selected.
6. Initial and boundary conditions necessary to solve elasticity theory equations are set.
7. Finite element dimensions are set.
8. Judging by computer resources and set tasks the most optimal configuration of accuracy and computational speed is selected.
9. Calculation results are treated.

Waveguide length is chosen considering minimum calculation expenses. As a rule, waveguide length is chosen to be equal to ultrasonic guided wave length, excited by frequency ω. Specified value is the smallest one in the set of frequencies, obtained as a result of solution of characteristic values task. This argument is derived from algorithm realization method.

Form of waveguide cross section was chosen in accordance with normative documents concerning recommended form of V-weld connection for gas main pipelines (Fig. 3) [9]. In future that will let us get a necessary instrument for the analysis of wavefield of ultrasonic guided wave, propagating in welded connections.

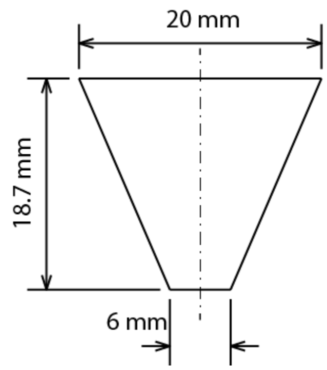


Figure 3 – Waveguide cross-section

Type of elastic medium is chosen out of welded connection material analysis (Table 1) [10].

Table 1 – Waveguide mechanical properties

Steel grade	Ст4
Young modulus, hPa	189
Poisson ratio	0.29
Yield point, MPa (σ_{02})	580
Material type	isotropic
Density, kg/m^3	7850

S.P. Timoshenko suggested mathematical model, describing elastic medium agitation while guided wave propagation. On the basis of this model, beam is a finite element type. For such types of finite elements algorithm of eigenvalues search can be used.

We choose zero initial conditions. To set boundary conditions it must be mentioned that wave is propagating along rod axis, which is being described as a finite element. In this case we build constrained equations on nodes that are on both ends of the waveguide that makes transitions and rotations equal. In this case on two ends of a waveguide, condition of equality of finite element nodes transition is set. To increase accuracy of calculations, waveguide length should be divided into 10 elements. Small number of elements let us conduct calculations with double accuracy.

Mode excitation can be evaluated by its effective mass $\mathbf{m}^{\mathbf{eff}}$ [11]. Effective mass calculation depends on normalization method and can be calculated by the following system of equations:

$$m_i^{\mathbf{eff}} = \hat{M}^{-1}(\psi_i \mathbf{M}\mathbf{T}_i)^2 , \qquad (26)$$

$$\mathbf{T_i} = \begin{bmatrix} 1 & 0 & 0 & 0 & \xi_3-\xi_3^0 & \xi_2^0-\xi_2 \\ 0 & 1 & 0 & \xi_3^0-\xi_3 & 0 & \xi_1-\xi_1^0 \\ 0 & 0 & 1 & \xi_2-\xi_2^0 & \xi_1^0-\xi_1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \hat{e},$$

where i is the number of mode; $\hat{M}$. is the generalized mass, equal to selected method of point normalization; ξ_j^0 is the coordinates of rotation centre; $\hat{e}$ is the single vector which points to form of mode motion, in which only one component is equal to 1 and the other five are equal to 0.

Effective mass approximation to waveguide mass means that mode can be excited in real waveguide, thus, this mode corresponds with the condition of ultrasonic guided wave propagation. From (27) it's clear that modes can be evaluated in accordance with oscillations form (displacement and spinning).

Calculations, carried out in two frequency ranges (Table 2, Table 3) show that with the increase of frequency algorithm finds equal number of modes in a more narrow frequency range.

Numerical results of found modes have been evaluated according to (27), (Fig. 4, Fig. 5). Obtained dependencies for effective mass show that only a couple of modes from found set correspond with the specified boundary conditions.

In megahertz frequency range it's seen that there are modes, which have definite twisting form of elastic medium elementary volume displacements. At lower frequencies twisting and axial displacements practically belong to one group of modes.

Table 2 – Results of calculations of ultrasonic guided wave propagation in semi-megahertz frequency range

Frequency, kHz	Characteristic value, $\times 10^{11}$
527	109
574	131
577	131
614	147
635	159
694	190
695	191
745	219
805	256
807	257
817	263
824	268
828	271
834	275
836	276

Table 3 – Results of calculations of ultrasonic guided wave propagation in megahertz frequency range

Frequency, kHz	Characteristic value, $\times 10^{11}$
1022	412
1037	425
1038	426
1053	437
1153	525
1155	526
1160	531
1214	582
1226	594
1232	599
1233	600
1238	601
1263	630
1272	639
1275	642

Conclusions

This paper has specified that parameters of ultrasonic guided waves propagation in a waveguide with cross section of finite dimensions can be calculated with the help of existing algorithms, which apply method of finite elements for the search of characteristic values like Timoshenko beam. It has been found out that as a result of characteristic values calculation with boundary conditions which correspond with set boundary conditions for specified wave number we also get characteristic values which are not responsible for modes propagation. The results of calculations must be filtrated on the basis of mode effective mass value criterion.

It has been investigated that mode effective mass value criterion can be used to determine type of guided wave mode. Modes have been found which propagate in waveguide with finite dimensions of its cross section and definite torsional displacements of elementary volume without additional axial displacements. Wave energy in such guided waves is mostly concentrated in torsional modes and doesn't go over to modes which can be propagating in plates (symmetrical and antisymmetrical modes).

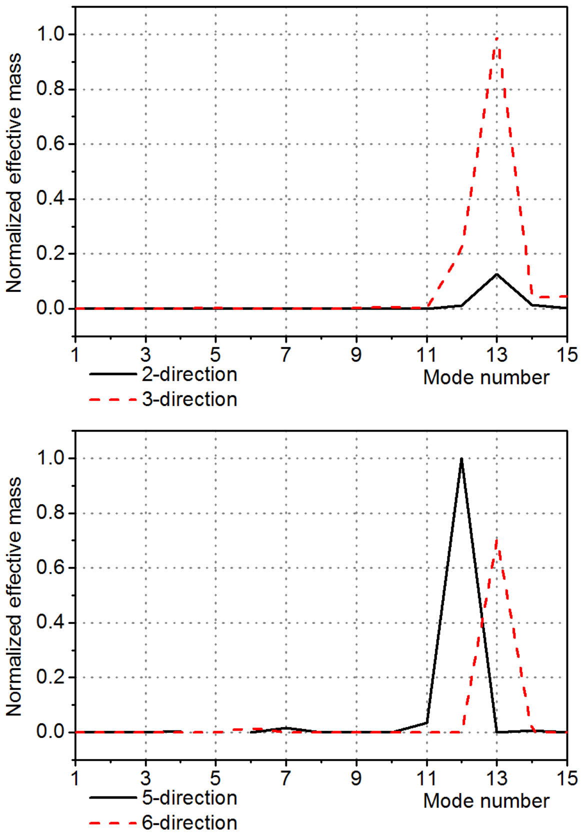


Figure 4 – Effective mass of waveguide modes in semi-megahertz frequency range

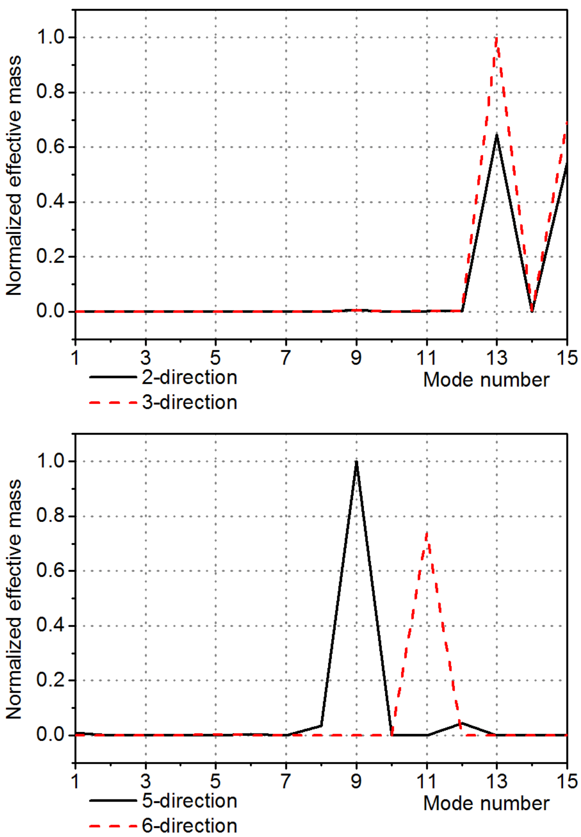


Figure 5 – Effective mass of waveguide modes in megahertz frequency range

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Моделювання поширення спрямованих ультразвукових хвиль е хвилеводі зі скінченним розміром його поперечного перерізу

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Актуальним завданням є подальший розвиток методів контролю об'єктів нафтогазової промисловості ультразвуковими спрямованими хвилями. Метою дослідження є розроблення математичної моделі поширення ультразвукових спрямованих хвиль у нафтогазопроводах, що виготовлені зі сталі.

Методом дослідження є комп'ютерне моделювання процесу поширення ультразвукової спрямованої хвилі у хвилеводі зі скінченними розмірами його поперечного перерізу.

Проведено обчислення поширення спрямованих хвиль у двох частотних діапазонах. Зі збільшенням частоти алгоритм знаходить однакову кількість мод у більш вузькому частотному діапазоні. Числові результати знайдених мод були оцінені за критерієм ефективної маси моди. За цим критерієм видно, що тільки кілька мод зі знайденого набору відповідають накладеним граничним умовам поширення ультразвукової спрямованої хвилі.

Результати приведено для поширення ультразвукових спрямованих хвиль у V-подібному зварному шві, геометричні розміри якого відповідають нормам з'єднань магістральних нафтогазопроводів.

Встановлено, що параметри поширення ультразвукових спрямованих хвиль у хвилеводі зі скінченними розмірами його поперечного перерізу можна обчислювати за допомогою існуючих алгоритмів, що реалізують метод скінченних елементів для пошуку власних значень на основі елемента типу стрижня Тимошенко. Виявлено, що в результаті обчислення алгоритмом власних значень із заданими граничними умовами, які відповідають поширенню спрямованих хвиль для заданого хвильового числа, ми одержуємо також власні значення, які не відповідають за поширення мод. Результати обчислень необхідно фільтрувати за критерієм величини ефективної маси моди. Досліджено, що критерій ефективної маси моди можна використовувати для визначення типу спрямованої хвилі. Знайдено моди, що поширюються у хвилеводі зі скінченними розмірами його поперечного перерізу з вираженими крутним зміщенням елементарного об'єму без додаткових осьових зміщень.

Ключові слова: *крутна мода, метод скінченних елементів, ультразвук, ультразвукова спрямована хвиля.*