DOI: 10.31471/2311-1399-2020-1(13)-1-7

Interference of torsion waves in the underground pipeline caused by the movement of the damaged foundation

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Received: 26.06.2020 Accepted: 30.06.2020

Abstract

In this article, we study the strength of underground pipelines, which are operated in difficult mining and geological conditions in area full of tectonic faults. In such seismically active areas, in addition to the pressure load of the transported product, the pipe is subjected to additional effects from the movements of the damaged foundation. When the movements are transient,a dynamic analysis of the behavior of structures must be carried out. The aim of the study is to develop a model to describe the non-stationary process of deformation of the pipeline on the damaged foundation, caused by the sudden mutual reversal of several fragments of the base around the axis of the pipe. The dynamics of the pipeline was investigated in a linear setting, modeling it with an infinite tubular rod. We consider blocks of a basis to be absolutely rigid; the behavior of a thin layer of soil backfill is described with the help of Winkler's hypothesis. The kinematics of mutual rotations of the base fragments is given by discontinuous functions from the axial coordinate. The strength of the pipeline is assessed by summing the standard and non-standard stresses, while the pipe is considered a torque-free shell. This approach makes it possible to assess the strength of the underground pipeline not by the external load from the soil, which is usually unknown, but by the kinematic parameters of the movements of the fault banks. An initial-boundary value problem for the differential equation of torsion with a discontinuous right-hand side has been formulated. Based on the analytical solution of the problem, the influence of the interference of torsion waves excited by sudden reversals of the foundation fragments around the axis of the pipe on the stress state of the pipeline under pressure has been studied. It has been established that the dynamic effects significantly depend on the structure of the breaking movements of the foundation and on the distance between the faults.

Keywords: *strength; sudden rotation of foundation fragments; torsion wave interference; underground pipeline.*

Introduction

The modern approach to the calculations of underground pipelines is as follows [1–4]. Internal pressure, temperature difference and elastic bending of the pipe during profiling and turns of the route are considered to be standard loads that are taken into account in the design. The stress state of the pipeline is usually studied on the basis of geometrically linear or linearized theory of rods taking into account the interaction with the soil, and its calculation for strength is carried out within the momentless theory of shells at allowable stresses or limit state. In particularly critical cases, estimates are being defined, using 2D or 3D models of the deformable body. This approach in one form or another is reflected in the regulations and is implemented in application packages that are operated by design organizations while forecasting the life of pipeline systems.

Operation of underground pipelines in areas of abnormal behavior of the base belongs to non-standard operating conditions and requires additional theoretical analysis and related engineering and geological monitoring. Usually the mechanical load on the pipeline in abnormal areas is difficult to predict. Therefore, researchers often limit the establishment of allowable loads on the pipe crossing the landslide or fault, using analytical models [5, 6] or numerical calculation methods [7–10]. To predict the safety of oil and gas pipelines in landslide hazard mountains, an alternative model of pipeline deformation in places of local destruction of the rock base has been developed [11, 12], which operates with kinematic parameters of damage – discontinuities of displacements and angles of rotation. An example of the application of this model to the calculation of the behavior of a pipeline with additional elms in the fault zone is the publication [13]. In [14, 15], this approach is extended to the problem of modal analysis of stationary vibration of pipelines on a block basis. As far as we know, the effects of sudden

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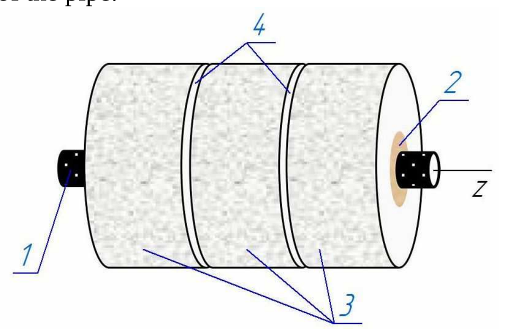
movements of the damaged foundation on the dynamics of underground pipelines have not yet been considered.

The purpose of this article is to develop a model to describe the non-stationary process of pipeline deformation on a damaged foundation caused by a sudden mutual reversal of several fragments of the base around the pipe axis. The rest of the article reflects the implementation of research objectives and is organized as follows. First, we present the key hypotheses of the model and formulate the corresponding initial-boundary value problem of dynamic torsion of the pipeline. Then we construct an analytical solution of the problem in the case of symmetrical and antisymmetrical perturbation and analyze the effect of torsion wave interference on the stress state of the pipe. Brief conclusions complete the publication.

Initial boundary value problem formuating

Let's consider a rectilinear pipeline, which is under normal internal pressure of the transported product and interacts with the block dense base through a layer of backfill (Fig. 1). At the initial moment of time the system is at rest, the pipe is loaded only with internal pressure. Later the fragments of the damaged foundation on both sides of the faults make movements that are considered given. In this article, in particular, we investigate the wave processes in the pipeline generated by the sudden rotations of the base blocks around the axis of the pipe.



1 – pipe, 2 – soil layer, 3 – base blocks, 4 – damage (fractures)

Figure 1 – Layout of the underground pipeline on a damaged foundation

Let's align the axis z with the axis of the pipe. Let the base be divided into three blocks by damages (fractures) localized at points $z = \pm a$.

Let us extend the main hypotheses of the model described in [11, 12] to the case of dynamic torsion of a pipeline on a damaged basis. This means: the analysis is performed in a geometrically and physically linear setting; the pipeline is modeled by an infinite rectilinear tubular rod; we consider blocks of a basis absolutely rigid; the behavior of a thin layer of backfill is described by the traditional Winkler's hypothesis; the kinematics of mutual rotations of the foundation fragments is given by discontinuous functions from the axial coordinate; the presence of the pipeline does not affect the specified kinematics of the base; the strength of the pipeline is assessed by summing the standard and non-standard stresses, while the pipe is considered a torque-free shell.

The initial-boundary value problem of the dynamic torsion of an infinite rod is formulated on the basis of these assumptions [16]. It includes the equation of motion of the rod taking into account the conjugation through the elastic layer:

$$\frac{\partial^2 \varphi_z}{\partial z^2} - \lambda^2 (\varphi_z - \varphi_z^0) = \frac{1}{c_2^2} \frac{\partial^2 \varphi_z}{\partial t^2},$$

$$-\infty < z < \infty, \quad t > 0, \tag{1}$$

initial conditions:

$$\varphi_z(z,0) = 0, \quad \frac{\partial \varphi_z}{\partial t}(z,0) = 0, \quad -\infty < z < \infty, \tag{2}$$

and boundary conditions at infinity:

$$\frac{\partial \varphi_z}{\partial z}(\pm\infty) = 0, \quad t > 0. \tag{3}$$

The following designations are accepted here:

z, t – axial coordinate and time;

$\varphi_z(z,t)$ – the angle of torsion of the pipeline;

$\lambda = \sqrt{\dfrac{\pi D^3 k_\tau}{4GJ_p}} = \sqrt{\dfrac{k_\tau}{Gh}}$ – earth restraint coefficient;

$c_2 = \sqrt{G/\rho}$ – shear wave propagation velocity;

GJ_p – rigidity of the pipe relative to torsion;

ρJ_p – the kinetic moment of inertia of the cross-section;

G, ρ – shear modulus and density of pipe material;

D, h – outer diameter and wall thickness of the pipe;

k_τ – shear stiffness coefficient of the soil bed;

$$\varphi_z^0(z,t) = \{\varphi_z^0(-\infty) + \Theta_1 H(z+a) +$$

$$+ \Theta_2 H(z-a)\} H(t) \tag{4}$$

predetermined base reversal function;

$\varphi_z^0(-\infty)$ – the angle of rotation of the extreme left block of the base;

Θ_1, Θ_2 – jumps of the angle of sudden rotation of the base blocks when crossing the faults $z = -a$ and $z = a$ correspondingly;

$H(\cdot)$ – the Heaviside step function,

$2a$ – distance between damages.

Analytical solution

First, let us consider the auxiliary problem of the propagation of waves of torsion from a single mutual reversal of blocks at the origin $z = 0$. Assume that

$$\varphi_z^0(z,t) = \frac{1}{2} \operatorname{sgn} z \, H(t). \tag{5}$$

Applying the method of the Laplace integral transform over time [17], we found an analytical solution of problem (1) – (3) for the perturbation postulated by expression (5):

$$\varphi_z^*(z,t)=\frac{1}{2}\{1-\cos(\lambda c_2 t)-$$
$$-\lambda^2\int_{|z|}^{c_2 t}\int_{|z|}^{c_2\tau} J_0(\lambda(\tau-\eta))J_0\left(\lambda\sqrt{\eta^2-z^2}\right)d\eta d\tau\times$$
$$\times H(c_2 t-|z|)\}\operatorname{sgn} z\,, \qquad (6)$$

where $J_0(t)$ is Bessel function of the first kind of zero order [18].

The angular velocity of rotation of the sections and the torsional deformation caused by a single perturbation were found by differentiating expression (5) according to time and coordinate, respectively.

$$\omega_z^*(z,t)=\frac{\partial\varphi_z^*}{\partial t}(z,t)=\frac{c_2}{2}\{\lambda\sin(\lambda c_2 t)-$$
$$-\lambda^2\int_{|z|}^{c_2 t} J_0(\lambda(\tau-\eta))J_0\left(\lambda\sqrt{\eta^2-z^2}\right)d\tau\times$$
$$\times H(c_2 t-|z|)\}\operatorname{sgn} z\,, \qquad (7)$$

$$\gamma_{z\theta}^*(z,t)=\frac{D}{2}\frac{\partial\varphi_z^*}{\partial z}(z,t)=$$
$$=\frac{D\lambda^2}{4}\int_{|z|}^{c_2 t} J_0\left(\lambda\sqrt{\tau^2-z^2}\right)d\tau\, H(c_2 t-|z|)\,. \qquad (8)$$

We now turn to the effects of interference of waves generated by two discontinuities of the base. Let's investigate symmetrical and antisymmetrical problems separately.

Symmetrical torsion

Suppose in the formula (4) $\varphi_z^0(-\infty)=0$, $\Theta_1=\Theta$, $\Theta_2=-\Theta$. Since, $H(z)=(1+\operatorname{sgn}(z))/2$, then

$$\varphi_z^0(z,t)=\frac{\Theta}{2}(\operatorname{sgn}(z+a)-\operatorname{sgn}(z-a))H(t)\,.$$

This means that the middle fragment of the base rotates at an angle Θ relative to the fixed extremities (Fig. 2).

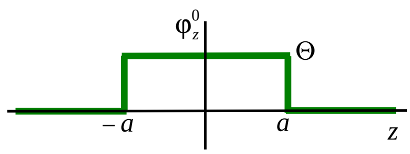


Figure 2 – The angle of rotation of the base blocks at $t>0$ (symmetric problem)

In this case, the solution of the problem is the superposition of the angles of rotation from two sources of perturbation:

$$\varphi_z(z,t)=\Theta\left(\varphi_z^*(z+a,t)-\varphi_z^*(z-a,t)\right). \qquad (9)$$

Similarly for angular velocity and angular deformation

$$\omega_z(z,t)=\Theta\left(\omega_z^*(z+a,t)-\omega_z^*(z-a,t)\right), \qquad (10)$$

$$\gamma_z(z,t)=\Theta\left(\gamma_{z\theta}^*(z+a,t)-\gamma_{z\theta}^*(z-a,t)\right). \qquad (11)$$

Antisymmetrical torsion

Let in the formula (4) $\varphi_z^0(-\infty)=-\Theta$, $\Theta_1=\Theta$, $\Theta_2=\Theta$. Then

$$\varphi_z^0(z,t)=\frac{\Theta}{2}(\operatorname{sgn}(z+a)+\operatorname{sgn}(z-a))H(t)\,.$$

This means that the two semi-infinite fragments of the base rotate in different directions at an angle Θ relative to the fixed middle block (Fig. 3).

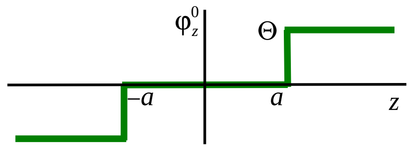


Figure 3 – Angle of rotation of the base blocks at $t>0$ (antisymmetrical problem)

The solution of the antisymmetric problem will be superposition

$$\varphi_z(z,t)=\Theta\left(\varphi_z^*(z+a,t)+\varphi_z^*(z-a,t)\right). \qquad (12)$$

In its turn

$$\omega_z(z,t)=\Theta\left(\omega_z^*(z+a,t)+\omega_z^*(z-a,t)\right), \qquad (13)$$

$$\gamma_{z\theta}(z,t)=\Theta\left(\gamma_{z\theta}^*(z+a,t)+\gamma_{z\theta}^*(z-a,t)\right). \qquad (14)$$

Thus, expressions (9) – (11) and (12) – (14) give a complete picture of the kinematics and deformation of the pipeline in cases of symmetrical and antisymmetrical problems. Due to the limited volume of publication, we will not analyze these fields, but will proceed immediately to a detailed study of the stress state of the pipe.

Analysis of dynamic stresses

The components of the stress tensor in the pipe wall are found from the relations:

$$\sigma_z=\nu p\frac{D}{2h},\ \sigma_\theta=p\frac{D}{2h},\ \tau_{z\theta}=G\gamma_{z\theta}\,. \qquad (15)$$

Here p is internal pressure in the pipeline; ν is Poisson's ratio of pipe material.

In formula (15), the first two expressions specify the normal stresses caused by the standard internal pressure p, and the third expression is the off-line tangential stress from the dynamic torsion. To calculate it, we use deformations (11) and (14).

The ultimate equilibrium of the pipe is estimated by the energy theory of strength [16]:

$$\sigma_{eq}\le[\sigma]\,,$$

where σ_{eq} is equivalent Mises stress

$$\sigma_{eq}=\sqrt{\sigma_z^2-\sigma_z\sigma_\theta+\sigma_\theta^2+3\tau_{z\theta}^2}\,; \qquad (16)$$

$[\sigma]$ is allowable stress for pipe material.

Numerical calculations were performed for the underground main pipeline with the following parameters: $D=1420$ mm, $h=18$ mm, $G=81000$ MPa, $\nu=0.3$, $\rho=7800$ kg/m^3. For backfilling used $k_\tau=2$ MPa/m. The internal pressure was considered to

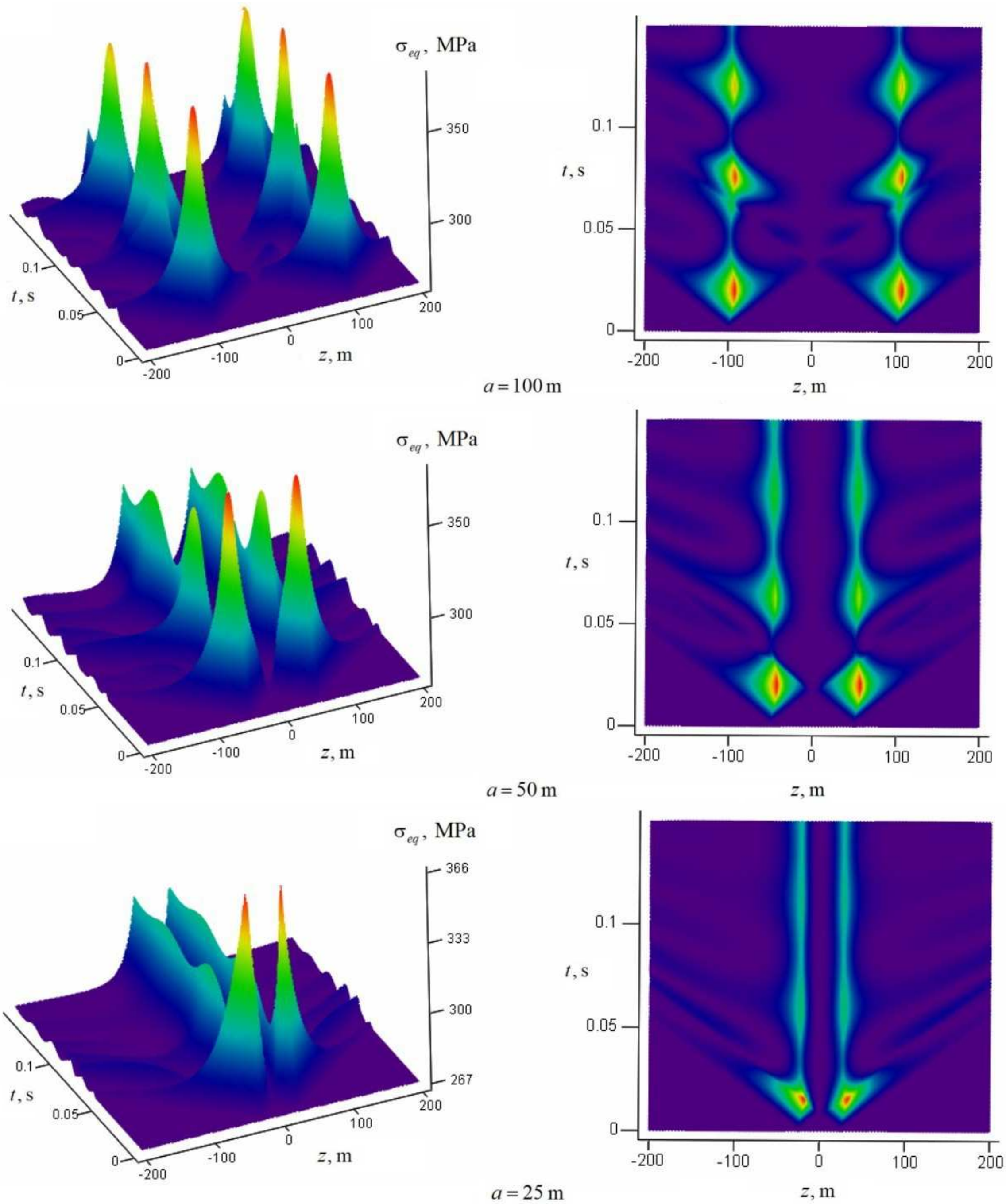


Figure 4 – Equivalent stresses in the pipe due to the sudden rotation of the middle fragment of the foundation for different distances between faults (symmetrical problem)

create a tangential stress $\sigma_\theta = 300\,\text{MPa}$ in the pipe. The angle of rotation of the foundation fragment around the axis of the pipe $\Theta = 0.1\,\text{rad}$. The values of the half-distance between the faults of the base varied: $a = 100$ m; 50 m; 25 m.

Using formulas (11), (15), (16) and (14), (15), (16), the graphs of unsteady fields of equivalent stress in the pipe wall for symmetrical (Fig. 4) and antisymmetrical (Fig. 5) problems were constructed. Striving for greater clarity, all drawings were made in spatial and flat versions. Already the first cursory analysis of these graphs shows that cross-sections of the pipe at the location of fractures are the most intense $z = \pm a$. Given this in Figs. 6, 7 dependences of equivalent stresses in cross sections $z = \pm a$ on time were additionally constructed. For comparison, on each graph, a dashed line shows a similar curve for an isolated single fault, constructed by formulas (15), (16) by deformation $\gamma_z(0,t) = \Theta\gamma^*_{z\theta}(0,t)$ (or the same as $a \to \infty$ in formulas (11) and (14)).

The presented graphic material shows that the effects of wave interference for symmetrical and antisymmetrical circles are fundamentally different. In the case of rotation of the middle block of the base, the convergence of faults generally leads to a decrease in stresses in the pipe (see Fig. 4). At the same time, with

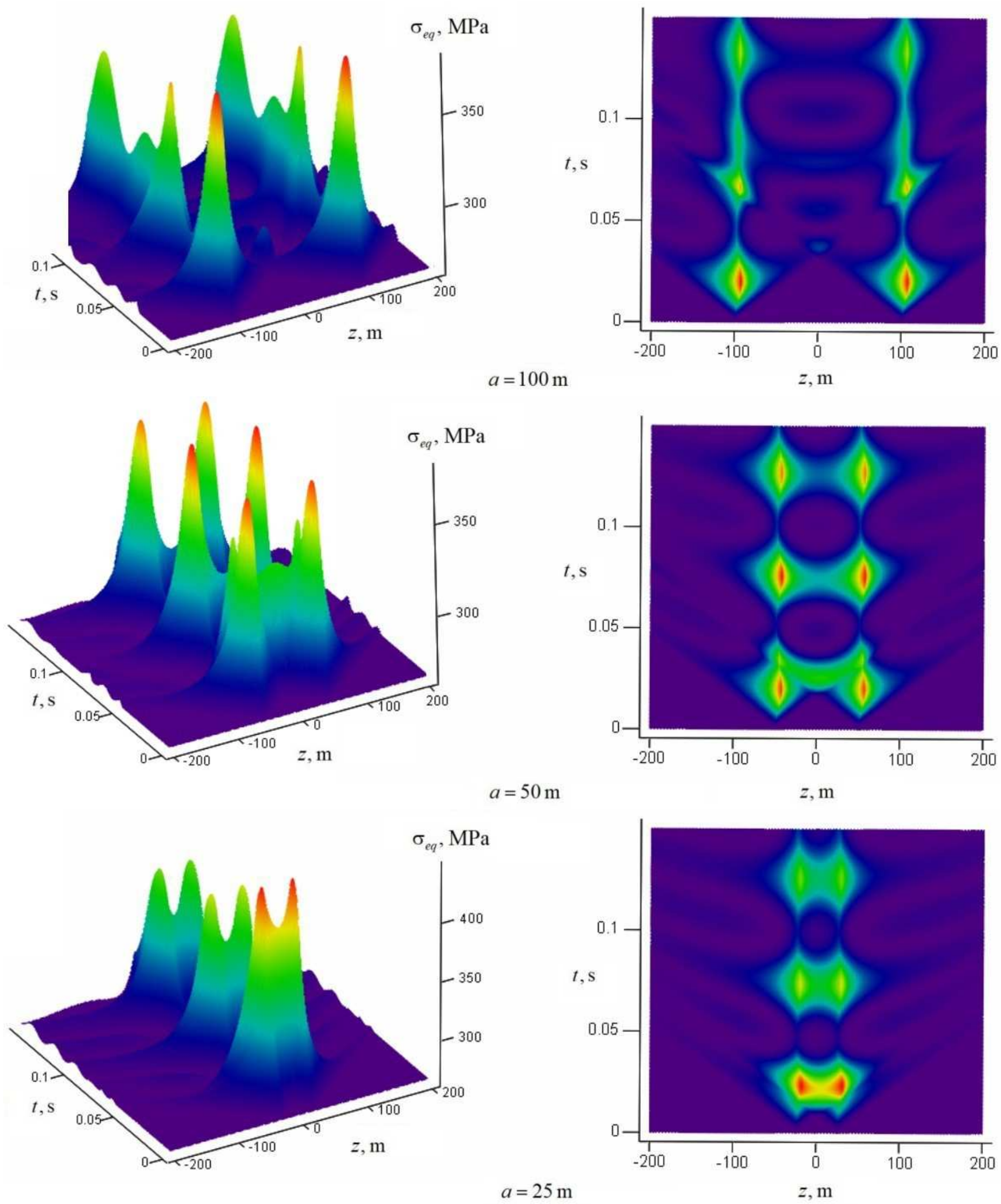


Figure 5 – Equivalent stresses in the pipe due to the sudden rotation of the extreme fragments of the foundation in the opposite direction for different distances between faults (antisymmetrical problem)

an asymmetrical reversal of the extreme blocks, the decrease in the distance between faults is accompanied by a significant increase in equivalent stresses (see Fig. 5). We explain such regularities by the fact that in the first case the interfered waves, emitted by discontinuities of turns, dampen each other, and in the second case on the contrary – mutually amplify.

Details of the change in stresses in the cross-sections of the pipe at the fault location are clearly visible on one-dimensional graphs (see Figs. 6, 7). Before the arrival of the wave from the adjacent damage to the foundation, the pipe oscillates as in the case of a single rupture of the angle of rotation of the base. In the case of a symmetrical problem, the arrival of a wave from an adjacent defect of the base does not change the value of the maximum stress ($\max\sigma_{eq} \approx 380$ MPa); for short distances, for example, for $a = 25$ m max σ_{eq} it may be even decreased (see Fig. 6). At the same time, in the case of an antisymmetrical problem, the interaction of torsion waves increases the maximum values σ_{eq}, especially since the distance between the faults is smaller (see Fig. 7).

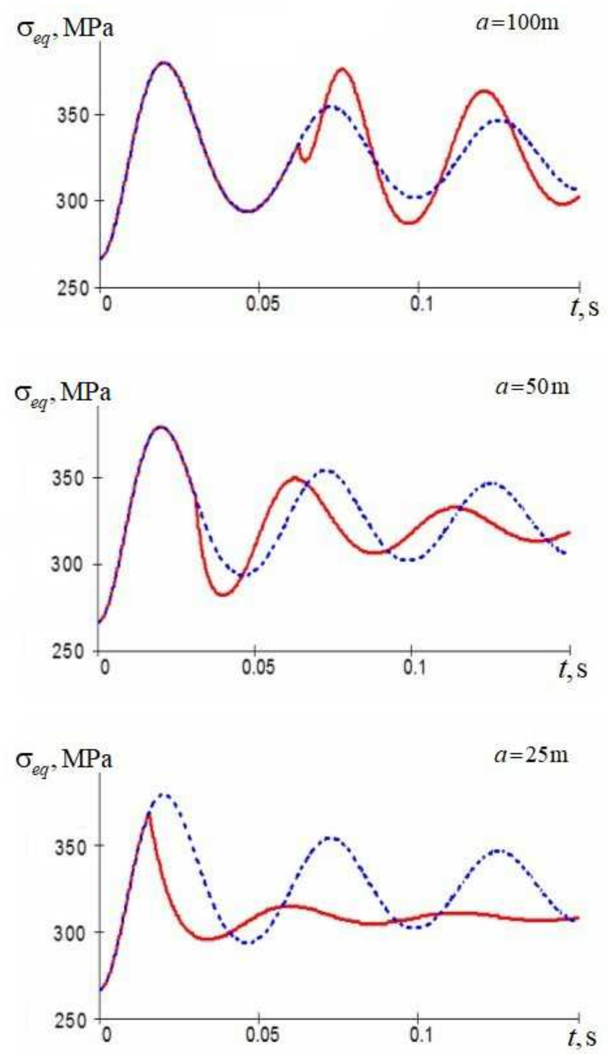


Figure 6 – Dependences of equivalent stress at the fracture location on time for different distances between faults (symmetrical problem)

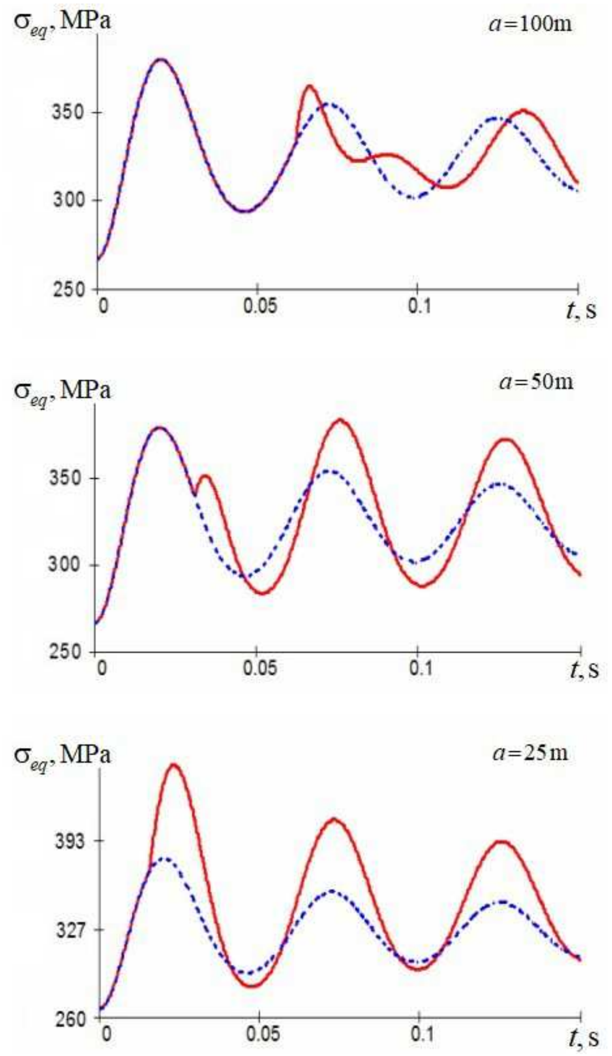


Figure 7 – Dependences of equivalent stress at the fracture location on time for different distances between faults (antisymmetrical problem)

Conclusions

A technique for the analysis of the deformation of the underground pipeline on the damaged foundation has been developed, which allows to estimate the kinematics and the stress-strain state of the pipe according to the given parameters of the nonstationary reversal of the foundation fragments.

The interference of torsion waves excited by mutual rotations of the foundation blocks at two faults has been investigated. It has been defined that the effect of the interaction of waves is to reduce the dynamic stresses in the symmetrical reversal of the foundation fragments and to increase the stresses in the case of antisymmetrical reversal. Quantitatively, these patterns are enhanced by reducing the distance between damages.

The research studies described in this article should be developed taking into account the inertia of the transported product and the non-elastic properties of the soil layer. The starting point for such a research can be the formulation of problems of the dynamics of rods with nonlinear conditions on the side surface, described in [19, 20].

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УДК 539.4; 622.692

Інтерференція хвиль скручення у підземному трубопроводі, збурених рухом пошкодженої основи

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У статті вивчено питання міцності підземних трубопроводів, які експлуатуються в складних гірничогеологічних умовах на територіях із тектонічними розломами. У таких умовах труба, окрім навантаження тиском транспортованого продукту, зазнає додаткових впливів від переміщень пошкодженої основи, що потребує динамічного аналізу поведінки конструкцій. Мета роботи полягає у розробці моделі для описання нестаціонарного процесу деформування трубопроводу на пошкодженій основі, спричиненого раптовим взаємним розворотом декількох фрагментів основи довкола осі труби. Динаміку трубопроводу досліджували в лінійній постановці, моделюючи його нескінченним трубчастим стержнем. Блоки основи вважаємо абсолютно жорсткими; поведінку тонкого шару ґрунтової засипки описуємо гіпотезою Вінклера. Кінематику взаємних поворотів фрагментів основи задано розривними функціями від осьової координати. Міцність трубопроводу оцінено сумою штатних від внутрішнього тиску та позаштатних напружень від скручування, при цьому трубу вважається безмоментною циліндричною оболонкою. Такий підхід дає можливість оцінювати міцність підземного трубопроводу не за зовнішнім навантаженням від ґрунту, яке зазвичай є невідомим, а за кінематичними параметрами рухів берегів розломів. Сформульовано початково-крайову задачу для диференціального рівняння скручування з розривною правою частиною. На підставі аналітичного розв'язку задачі вивчено вплив інтерференції хвиль скручування, збурених раптовими розворотами фрагментів основи довкола осі труби на напружений стан трубопроводу під тиском. Встановлено, що динамічні ефекти істотно залежать від структури розривних рухів основи та дистанції між розломами.

Ключові слова: *інтерференція хвиль скручування; міцність; підземний трубопровід; раптовий поворот фрагментів основи.*