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Determination of the rheological properties of drilling fluids from rotational viscometry data

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Abstract

The method of rotational viscometry data processing, which is based on the maximum likelihood function principle is considered. The method takes into account the informative content of experiments and is built on the strict solution of the Couette flow equation in a viscosimeter gap. The class of models is formed from rheologically stationary (including viscosity) models. A generalization of the model for processing the experimental plan data with the aim of building equations of the state of the rheological properties of variable factors is considered. A multicriterial interpretation of estimates of the rheological model and properties of liquids has been proposed. Illustrative examples of estimating rheological properties building of drilling fluids and their equations of state are given

Keywords: *biviscosity fluids, Couette flow, equation of state, maximum principle of likelihood function, multi-criteria analysis, rheologically stationary model.*

Introduction

The basis of modern technologies for the construction of oil and gas wells is the use of drilling fluid systems, the composition and properties of which determine the effectiveness of technological operations.

The rheological properties of drilling fluids characterize the most important functional requirements for technology, due to the dynamic characteristics of fluid flows in the elements of the well circulation system [1, 2]. This requires a series of studies, as well as monitoring and regulation of the rheological properties of drilling fluids in order to select the optimal parameters of well drilling technologies. The latter is based on reliable information on drilling fluids rheological properties and has a significant impact on well construction performance [3–7].

The most common rheological models of drilling fluids are as follows: $\dot{\gamma}(\tau)=\tau/\eta$ (Newton 1686 [8]), $\dot{\gamma}(\tau)=(\tau-\tau_0)/\eta$ (Bingham [9]), $\dot{\gamma}(\tau)=(\tau/k)^{1/n}$ (Ostwald [10]), $\dot{\gamma}(\tau)=\left((\tau-\tau_0)/k\right)^{1/n}$ (Herschel and Bulkley [11]), $\dot{\gamma}(\tau)=\left(\tau^{1/n}-\tau_0^{1/n}\right)^n/\eta$ (Schulman [12]), $\dot{\gamma}(\tau)=(\tau/k)^{1/n}-\beta$ (Robertson and Stiff [13]) and etc., where $\dot{\gamma},\ \tau$ are shear rate and shear stress; $\eta,\ \tau_0,\ k,\ n,\ \beta$ are rheological properties. The objectivity of information on rheological properties is closely related to an increase in the number of parameters of rheological models.

Generalization of rheological models for biviscosity fluids was suggested by Myslyuk and Salyzhyn [14]

$$\dot{\gamma}(\tau)=\begin{cases}\dot{\gamma}\left(\tau,a^{(1)}\right),\tau<\tau^*;\\ \dot{\gamma}\left(\tau,a^{(2)}\right),\tau\geq\tau^*,\end{cases} \tag{1}$$

where $a^{(1)}$, $a^{(2)}$ are rheological properties, respectively, for low and high shear rate gradients; τ^* is boundary shear stress corresponding to one of the real nontrivial roots of the equation $\dot{\gamma}\left(\tau^*,a^{(1)}\right)=\dot{\gamma}\left(\tau^*,a^{(2)}\right)$. Models (1) allow using arbitrary combinations of the above mentioned models and, in terms of piecewise approximation, expand the interpretation of measurement results.

A rheological model with a logarithmic correction coefficient was considered Nasiri and Ashrafizadeh [15]

$$\tau=\tau_0+b\ \dot{\gamma}+k\ \dot{\gamma}^{n-m}\ \ln(1+\dot{\gamma}), \tag{2}$$

where $\tau_0\geq 0,\ b\geq 0,\ 0\leq m\leq 1,\ 0\leq n\leq 1$.

Bui and Tutuncu [16] suggested rheological model in the form of cubic splines

$$\tau_j=c_{0j}+c_{1j}\dot{\gamma}+c_{2j}\dot{\gamma}^2+c_{3j}\dot{\gamma}^3,\ j=1,2,..., \tag{3}$$

where c_j is a parameter vector of j spline.

Andaverde et al. [3] suggested log-linear method, based on the rheological model

$$\tau=\frac{a\ c-b}{c^2}\ \ln(1+c\dot{\gamma})+\frac{b}{c}\dot{\gamma}+\tau_0, \tag{4}$$

where $a,\ b,\ c,\ \tau_0$ are rheological properties, $\tau_0\geq 0$.

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Currently, the rheological properties of drilling fluids are measured primarily using rotational viscometers with coaxial cylinders. The processing of viscometric data in the general case is reduced to the selectionac cording to the measurement results $\tau = \{\tau_i\}$ and $\omega = \{\omega_i\}$, $i = \overline{1,N}$ (where N is the number of speeds of rotation of the outer cylinder) of model $\hat{\nu}$ studied fluid from a certain a priori known class ϑ and building of rheological properties estimates $\hat{a}$.

The features of the interpretation of rotational viscometry data using the methodology Myslyuk [17] and its generalizations [14, 18, 19] are discussed below.

Evolution of rheological properties

Model processing

The theoretical basis for the interpretation of rotational viscometer data is the Couette flow equation in the measuring gap of a rotational viscometer

$$\omega = \frac{1}{2}\int_{\tau_c}^{\tau} \frac{\dot{\gamma}(\xi)}{\xi}\, d\xi, \qquad (5)$$

where ω is angular velocity of outer cylinder rotation; τ, τ_c are shear stress on the inner and outer cylinders;

$$\tau_c = \begin{cases} \alpha^2\tau, \text{ if } \tau \ge \tau/\alpha^2; \\ \tau_0, \quad \text{if } \tau = \left[\tau_0, \tau_0/\alpha^2\right[; \end{cases} \qquad (6)$$

$\alpha = R_b/R_c$; R_b, R_c are radii of the inner and outer cylinders; τ_0 is fluid yield stress.

For biviscosity fluids (1), the basic equation of rotational viscometry (5) is represented as

$$\omega = \begin{cases} \frac{1}{2}\int_{\tau_c}^{\tau} \frac{\dot{\gamma}\left(\xi, a^{(1)}\right)}{\xi} d\xi, \text{ if } \tau \le \tau^*; \\ \frac{1}{2}\left[\int_{\tau_c}^{\tau^*} \frac{\dot{\gamma}\left(\xi, a^{(1)}\right)}{\xi} d\xi + \int_{\tau^*}^{\tau} \frac{\dot{\gamma}\left(\xi, a^{(2)}\right)}{\xi} d\xi\right], \\ \qquad \text{if } \tau^* < \tau \le \tau^*/\alpha^2; \\ \frac{1}{2}\int_{\tau_c}^{\tau} \frac{\dot{\gamma}\left(\xi, a^{(2)}\right)}{\xi} d\xi, \text{ if } \tau > \tau^*/\alpha^2. \end{cases} \qquad (7)$$

Here, the representative flows in the viscometer gap are highlighted: for fluids $\dot{\gamma}\left(\tau, a^{(1)}\right)$ at $\tau < \tau^*$; for fluids $\dot{\gamma}\left(\tau, a^{(1)}\right)$ and $\dot{\gamma}\left(\tau, a^{(2)}\right)$ at $\tau^* < \tau < \tau^*/\alpha^2$; for fluids $\dot{\gamma}\left(\tau, a^{(2)}\right)$ at $\tau < \tau^*/\alpha^2$. Note that $\tau_0^{(1)}$ is used for biviscosity fluid in equation (6).

Assuming that the discrepancy between the vector of experimental measurements of shear stresses τ and its theoretical field $A(\omega, a_\nu)$ is additive, the task of rotational viscometry data processing is formalized in the form of [17]

$$\tau = \begin{cases} \text{either } A(\omega, a_1) + \varepsilon_1, \\ \ldots\ldots\ldots\ldots\ldots\ldots, \\ \text{or} \quad A(\omega, a_\nu) + \varepsilon_\nu, \ \nu \in \vartheta, \\ \ldots\ldots\ldots\ldots\ldots\ldots, \end{cases} \qquad (8)$$

where ε_ν is vector of a random component due to measurement errors with the normal distribution law of their probabilities. With respect to the vector ε_ν we will consider it centered (with zero mathematical expectation) with an unknown or known covariance matrix C.

Single processing

To solve problem (8), in a certain class ϑ of rheological models, the maximum likelihood function method was used, according to which firstits rheological properties $\hat{a}_\nu$ are estimated for each rheological model and then the mostad equate $\hat{\nu}$ rheological modelis defined [17]. Decision rules are equivalent to procedures for the known matrix

$$\min\left\|C^{-1/2}\left(\tau - A(\omega, a_\nu)\right)\right\| \Rightarrow \left(\hat{a}_\nu, \hat{\nu}\right) \qquad (9)$$

and unknown covariance matrix $C = \sigma_\nu^2 I$

$$\begin{cases} \min\left\|\tau - A(\omega, a_\nu)\right\| \Rightarrow \hat{a}_\nu, \nu \in \vartheta; \\ \min\left\{\sigma_{a\nu}^2 = \frac{1}{N - r_\nu}\sum_{i=1}^{N}\left[\tau_i - A(\omega, \hat{a}_\nu)\right]^2\right\} \Rightarrow \hat{\nu}, \end{cases} \qquad (10)$$

where $\hat{\sigma}_{a\nu}^2$, r_ν is assessment of adequacy variance and the number of evaluated parameters ν of the rheological model; while I is identity matrix.

For biviscosity models (1) procedures (9) and (10) are supplemented with a system of constraints for choosing points L and $L+1$, determining the boundary shear stress τ^* [14, 19]

$$\begin{cases} L \ge r^{(1)} + 1; \\ N - L \ge r^{(2)} + 1; \\ \tau_L \le \tau^* \le \tau_{L+1}, \end{cases} \qquad (11)$$

where $r^{(1)}, r^{(2)}$ are number of estimated model parameters for low and high shear rates.

The matrix of covariance estimates of the parameters of the rheological model $\hat{\nu}$ is determined by the inverse of the Fisher information matrix

$$O = \left(A'^*(\omega, \hat{a}_\nu) C^{-1} A'(\omega, \hat{a}_\nu)\right)^{-1}, \qquad (12)$$

where $A'(\omega, \hat{a}_\nu)$ is derivative matrix; A'^* – transponsed matrix A'. In the case of biviscosity fluids, the covariance matrix of parameter estimates is constructed using (12) separately for models with low $O^{(1)}$ and high $O^{(2)}$ shear rates.

Batch processing

In practice, it is often necessary to evaluate the rheological properties of drilling fluids for some experimental design X (Batch processing), for example, to study the influence of concentrations of reagents, conditions (pressure, temperature) and other factors. Obviously, such estimates are meaningful for the most adequate rheological model $\hat{\nu}$ under the experimental design X [18, 19].

In batch processing of data, the procedures for evaluating rheological properties $\hat{a}_{\nu}^{(j)}$ for models $\nu \in \vartheta$ at each j – th point of the experimental design X are similar to (9) and (10). The selection of the most appropriate rheological model $\hat{\nu}$ is carried out using generalized criteria for a known matrix C

$$\min \sum_{j=1}^{M} \left\| C^{-1/2} \left(\tau^{(j)} - A(\omega, \hat{a}_{\nu}) \right) \right\| \Rightarrow \hat{\nu} \qquad (13)$$

and unknown covariance matrix $C = \sigma_{\nu}^2 I$

$$\min \left\{ \sigma_{a\nu}^2 = \frac{1}{M(N - r_{\nu})} \sum_{j=1}^{M} \sum_{i=1}^{N} \left[\tau^{(j)} - A(\omega, \hat{a}_{\nu}) \right] \right\} \Rightarrow \hat{\nu}, \qquad (14)$$

where $\tau^{(j)}$ is vector of measurements of shear stresses at the j – th point of experimental design; M is the number of points in the experimental design.

Result analysis

Rheological properties

From (5) and (6), in particular, it follows that for the Newton model, the shear rate gradient depends on the angular velocity ω and the relative gap α of the viscometer

$$\dot{\gamma} = 2\omega / (1 - \alpha^2), \qquad (15)$$

and for other models – additionally from the rheological properties a_{ν} of liquid $\dot{\gamma} = \dot{\gamma}(\omega, \alpha^2, a_{\nu})$. It was previously noted [19–23] that the application of rotational viscometry data processing techniques using the formula for the shear rate gradient of viscous fluids (15) leads to inaccurate estimates of the rheological properties of non-Newtonian fluids, which depend on α, a_{ν} and can be significant.

Let us consider the influence of the relative gap of a rotational viscometer on the accuracy of estimates of the rheological properties of a liquid using procedure (10) and using formula (15) for the shear rate gradient of a Newtonian fluid. Table 1 displays the design and results of numerical experiments for the Herschel–Balkley fluid with the properties of $\tau_0 = (1,\ 4,\ 7,\ 10)$ Pa, $k = (0.1,\ 0.5,\ 0.9,\ 1.3)$ $\text{Pa} \cdot s^n$, $n = (0.40,\ 0.65,\ 0.90,\ 1.15)$ and relative gaps of a OFITE 800 rotational viscometer $\alpha = (0.468,\ 0.666,\ 0.8681,\ 0.9365)$.

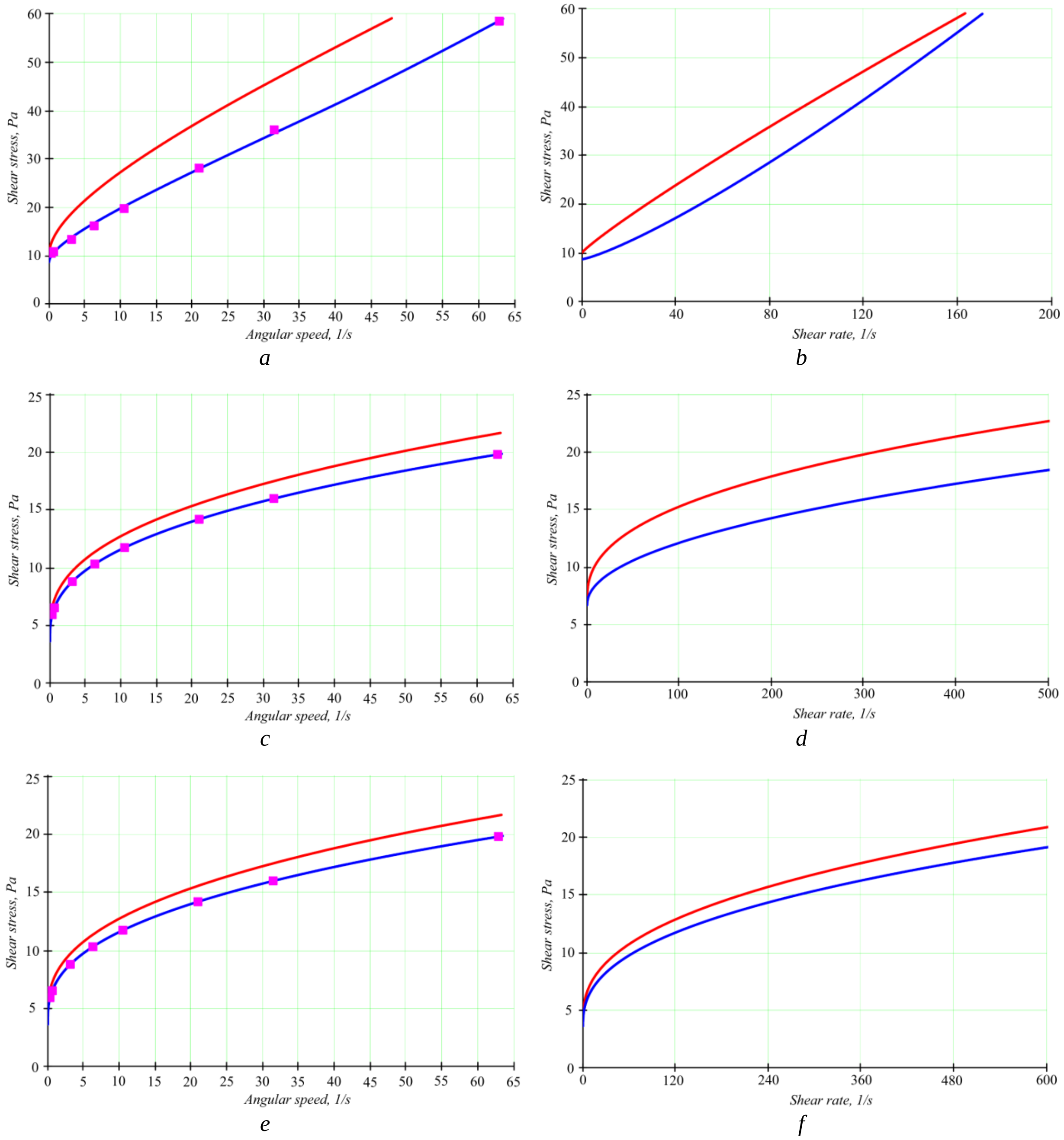
For the combinations of relative gaps and rheological properties shown in Table 1, the shear stresses τ_{γ} were calculated at the rotational speeds $\omega = (3,\ 6,\ 30,\ 60,\ 100,\ 200,\ 300,\ 600)$ min^{-1} of viscometer, which determine their exact values according to the method based on the use of formula (15). Note that, by virtue of (5) and (6), the values τ_{γ} are inaccurate for the Herschel–Balkley fluid.

Table 1 shows the estimates of the rheological properties $\hat{a}$ of the Herschel–Balkley fluid, constructed using procedure (10) based on τ_{γ} and ω. An analysis of the relative errors $\delta = 100 |a - \hat{a}| / \hat{a}$ in the estimates a of rheological properties using formula (15) indicates their significant values, especially for large annular gaps.The maximum errors for the dynamic shear stress is 83.32 %, the consistency measure is 1613.01 % and the non-linearity index is 39.02 %.

Figure 1 shows the rheograms and rheological curves of the Herschel–Balkley fluid for some points of the design of the numerical experiment (Table 1), which correspond to the rheological properties $\hat{a}$ and a. These results clearly illustrate the characteristic

Table 1 – Estimates results analysis of rheological properties

Test	Initial data				Estimates $\hat{a}$			Inaccuracies, %				
	α	τ_0, Pa	k, Pa·s^n	n	$\hat{\tau}_0$, Pa	$\hat{k}$, Pa·s^n	$\hat{n}$	δ_{τ_0}	δ_k	δ_n	δ_{τ}^{max}	$\delta_{\hat{\tau}}^{max}$
1	0.468	4	0.9	1.15	2.182	0.9664	1.1540	83.32	6.87	0.35	45.68	9.24
2	0.468	10	0.5	0.90	8.607	0.0924	1.2260	16.18	441.13	26.59	42.85	5.15
3	0.468	7	1.3	0.65	5,913	0.5714	0.7650	18.38	127.51	15.03	39.32	2.35
4	0.468	10	0.9	0.65	9.339	0.1575	0.9133	7.08	471.43	28.83	39.42	1.17
5	0.666	7	1.3	0.40	6.648	0.5903	0.4810	5.29	120.23	16.84	25.10	0.23
6	0.666	1	1.3	0.90	0.687	1.2560	0.9000	45.58	3.50	0	20.37	0.17
7	0.666	10	0.9	0.40	9.820	0.2434	0.5342	1.83	269.76	25.12	24.79	0.11
8	0.666	7	0.1	1.15	5.451	0.0755	1.2120	28.42	32.45	5.12	31.17	10.79
9	0.8681	1	0.5	1.15	0.840	0.5108	1.1500	19.05	2.11	0	4.87	1.33
10	0.8681	10	0.1	0.65	9.914	0.0058	1.0660	0.87	1613.01	39.02	11.73	1.11
11	0.8681	4	1.3	0.40	3.578	1.1830	0.4020	11.79	9.89	0.50	11.04	0.06
12	0.8681	4	0.1	1.15	3.483	0.1020	1.1500	14.84	1.96	0	12.91	0.48
13	0.9365	1	0.9	0.65	0.937	0.8802	0.6500	6.68	2.25	0	3.47	0.01
14	0.9365	4	0.1	0.90	3.752	0.0993	0.9001	6.61	7.05	0.01	6.02	0.02
15	0.9365	7	0.5	0.40	6.658	0.4592	0.4054	5.14	8.89	1.33	5.55	0.06
16	0.9365	1	0.5	0.90	0.938	0.4967	0.9000	6.57	0.66	0	2.55	0.02



a, *b* – test 2; *c*, *d* – test 5; *e*, *f* – test 11;
processing results using the formula (15); processing results using procedure (10);
simulated measurement data τ_γ and ω for the OFITE 800 viscometer

Figure 1 – Rheograms (*a*, *c*, *e*) and rheological curves (*b*, *d*, *f*) of Herschel–Balkley fluid according to Table 1

deviation of rheograms and curves for indicators a from the initial data (τ_γ and ω), as well as possible errors.

Table 1 also shows the maximum errors $\delta_\tau^{\max}$ and $\delta_{\hat{\tau}}^{\max}$ of shear stresses calculated using equations (5) and (6) at the points τ_γ and ω for evaluating the rheological properties, respectively. For small annular gaps $(\alpha = 0.9365)$, the errors $\delta_\tau^{\max}$ in the estimates of shear stresses are 2.55–6.02 %. Large values of the errors $\delta_{\hat{\tau}}^{\max}$ of shear stresses are related with lowest rotation speed $\omega = 3\ \text{min}^{-1}$.

In this regard, estimates of the parameters of rheological models in the form of cubic splines (3) [16], as well as with a logarithmic correction coefficient (2) [15] and (4) [3, were constructed using formulas for viscous fluid shear rate gradient. Let us consider the results of such estimates of rheological properties for models (3) and (4) in comparison with procedure (10).

Table 2 – Processing data results of rotational viscometry by Myslyuk [17] and Bui and Tutuncu [16]

Measurement results		Estimated Values					
ω, min^{-1}	τ, Pa	$\hat{\tau}_a$, Pa	δ_a, %	$\hat{\tau}_{cs}$, Pa	δ_{cs}, %	$\hat{\tau}_{GB}$, Pa	δ_{GB}, %
Data from Table 3 (Bui and Tutuncu [16])							
3	4.09	4.40	7.58	4.29	4.89	3.83	6.36
6	5.11	5.33	4.31	5.32	37.96	5.04	1.37
100	17.37	16.37	5.76	18.31	5.41	18.21	4.83
200	23.51	23.95	1.87	21.15	10.04	25.65	9.10
300	29.64	30.16	1.75	30.41	2.60	31.43	6.04
600	47.01	46.93	0.17	47.91	1.91	44.60	5.13
$\sum_i (\tau_i - \hat{\tau}_i)^2$, Pa^2		1.60		2.00		14.37	
Data from Table 4 (Bui and Tutuncu [16])							
3	0.958	1.031	7.62	0.983	2.61	1.096	14.41
6	1.437	1.313	8.63	1.491	3.76	1.364	5.08
100	6.226	6.409	2.94	6.411	2.97	6.209	0.27
200	10.537	10.390	1.40	10.724	1.77	9.994	5.15
300	13.890	13.890	0.00	14.175	2.05	13.320	4.10
600	22.990	23.020	0.13	23.381	1.70	22.000	4.31
$\sum_i (\tau_i - \hat{\tau}_i)^2$, Pa^2		0.08		0.32		1.62	

Table 3 – Assessment of rheological properties

Model ν	Rheological properties $\hat{a}_\nu$	Variance of adequacy $\hat{\sigma}^2_{a\nu}$, Pa^2
Data from Table 3 (Bui and Tutuncu [16])		
Bingham, 1	$\hat{\tau}_0 = 6.239$ Pa, $\hat{\eta} = 0.0417$ Pa$\cdot$s	9.8410
Ostwald, 2	$\hat{k} = 0.9223$ Pa$\cdot s^n$, $\hat{n} = 0.5595$	2.7660
Herschel–Bulkley, 3	$\hat{\tau}_0 = 2.948$ Pa, $\hat{k} = 0.4062$ Pa$\cdot$s^n, $\hat{n} = 0.6714$	0.8107
Schulman–Casson, 4	$\hat{\tau}_0 = 1.73$ Pa, $\hat{\eta} = 0.0126$ Pa$\cdot$s, $\hat{n} = 3.037$	0.5336
Data from Table 4 (Bui and Tutuncu [16])		
Bingham, 1	$\hat{\tau}_0 = 1.821$ Pa, $\hat{\eta} = 0.0216$ Pa$\cdot$s	1.3740
Ostwald, 2	$\hat{k} = 0.1664$ Pa$\cdot s^n$, $\hat{n} = 0.7079$	0.1397
Herschel–Bulkley, 3	$\hat{\tau}_0 = 0.5833$ Pa, $\hat{k} = 0.1175$ Pa$\cdot$s^n, $\hat{n} = 0.7553$	0.0255
Schulman–Casson, 4	$\hat{\tau}_0 = 0.8262$ Pa, $\hat{\eta} = 0.0149$ Pa$\cdot$s, $\hat{n} = 1.98$	0.2295

Table 2, based on data Bui and Tutuncu [16], shows the results of measurements τ_i and ω_i, $i = \overline{1,6}$ on a FANN 35 rotational viscometer (relative clearance α=0.9365), as well as data processing using (10) and model (3). Estimates of the rheological properties in accordance with (8) are constructed for an unknown stationary uncorrelated covariance matrix $C = \sigma_\nu^2 I$. Here the class ϑ is represented by models of non-Newtonian fluids: Bingham (ν=1), Ostwald (ν=2), Herschel–Balkley (ν=3) and Schulman–Casson (ν=4).

Table 3 shows the results of evaluations of rheological properties $\hat{a}_\nu$ using the first procedure (10) and adequacy variances $\hat{\sigma}^2_{a\nu}$. The most appropriate rheological models for data Bui and Tutuncu [16] in accordance with the second procedure (10) are the Shulman–Casson ν=4 and Herschel–Balkley $\hat{\nu}$=3 models.

From the covariance matrices O of the estimates of the models rheological properties indicators (12), we have their statistical characteristics (standard deviations, correlation coefficients):

σ_{τ_0}, Pa	0.9446/0.129;
$r_{\tau_0\eta} / r_{\tau_0 k}$	0.9354/–0.739;
σ_η / σ_k, $Pa \cdot s / Pa \cdot s^n$	0.0055/0.0113;
$r_{\tau_0 n}$	–0.9785/0.694;
σ_n	0.6756/0.0129;
$r_{\eta n} / r_{kn}$	–0.9877/–0.996.

Here, the numerator shows the characteristic values for the data of Table 3 Bui and Tutuncu [16], and the denominator indicates the data for Table 4 Bui and Tutuncu [16]. The indices in the characteristics indicate their compliance with the rheological properties.

Table 4 – Data processing results of rotational viscometry according to Andaverde et al. [3] and Myslyuk [17]

Measurement results		Estimated values $\hat{\tau}$ and δ			
ω, min^{-1}	τ, Pa	$\hat{\tau}_a$, Pa	δ_a, %	$\hat{\tau}_{LL}$, Pa	δ_{LL}, %
Series KE-11 (Kelessidis et al. [24])					
3	0.50	0.48	4.0	0.61	22.2
6	0.58	0.70	20.7	0.72	24.8
10	1.00	0.94	6.0	0.87	12.9
20	1.33	1.40	5.3	1.23	7.8
30	1.50	1.76	17.3	1.56	4.3
60	2.42	2.61	7.9	2.49	2.9
80	2.92	3.07	5.1	3.04	4.2
100	3.42	3.49	2.0	3.55	3.8
200	5.58	5.17	7.3	5.60	0.3
300	6.83	6.51	4.7	7.07	3.4
400	7.92	7.67	3.2	8.17	3.1
500	8.58	8.71	1.5	9.00	4.9
600	9.33	9.66	3.5	9.65	3.4
σ_a^2, Pa^2		0.0553		0.0554	
Series KE-16 (Kelessidis and Maglione [25])					
3	17.67	17.81	0.8	18.15	2.7
6	17.33	18.18	4.9	18.52	6.9
60	20.58	20.83	1.7	21.60	4.9
100	22.00	22.17	1.1	23.25	5.7
200	25.17	24.96	0.8	26.51	5.3
300	27.75	27.37	1.4	29.06	4.7
600	33.33	33.51	0.5	34.87	4.6
σ_a^2, Pa^2		0.3228		3.3828	

Table 5 – Assessment of rheological properties

Model ν	Rheological properties $\hat{a}_\nu$	Variance of adequacy $\hat{\sigma}_{av}^2$, Pa^2
Series KE-11 (Kelessidis et al. [24])		
Bingham, 1	$\hat{\tau}_0 = 1.2$ Pa, $\hat{\eta} = 0.009$ Pa·s	0.5756
Ostwald, 2	$\hat{k} = 0.1829$ Pa·s^n, $\hat{n} = 0.5685$	0.0553
Herschel–Bulkley, 3	$\hat{\tau}_0 = 0.0001$ Pa, $\hat{k} = 0.1839$ Pa·s^n, $\hat{n} = 0.5677$	0.0608
Schulman–Casson, 4	$\hat{\tau}_0 = 0.4758$ Pa, $\hat{\eta} = 0.0039$ Pa·s, $\hat{n} = 2.51$	0.2079
Series KE-16 (Kelessidis and Maglione [25])		
Bingham, 1	$\hat{\tau}_0 = 17.43$ Pa, $\hat{\eta} = 0.0156$ Pa·s	1.2290
Ostwald, 2	$\hat{k} = 11.77$ Pa·s^n, $\hat{n} = 0.1298$	4.8590
Herschel–Bulkley, 3	$\hat{\tau}_0 = 15.93$ Pa, $\hat{k} = 0.1867$ Pa·s^n, $\hat{n} = 0.6431$	0.1020
Schulman–Casson, 4	$\hat{\tau}_0 = 16.13$ Pa, $\hat{\eta} = 0.0074$ Pa·s, $\hat{n} = 1.4910$	0.1955

The calculated values of shear stresses τ_a were performed using the basic equation of rotational viscometry (5) and (6) for the most adequate model and are shown in Table 2. Similarly, using (5) and (6), shear stresses $\hat{\tau}_{cs}$ were calculated for spline models (3), the parameters of which are given in [16].

Table 2 also shows the results of calculations of shear stresses $\hat{\tau}_{GB}$ according to equations (5) and (6) for evaluating the rheological properties of Herschel – Balkley fluids obtained in [16] using formulas for the shear rate gradient of a viscous fluid: $\hat{\tau}_0 = 1.00$ Pa, $\hat{k} = 1.15$ Pa·s^n, $\hat{n} = 0.52$ (Table 4 data); $\hat{\tau}_0 = 0.662$ Pa, $\hat{k} = 0.119$ Pa·s^n, $\hat{n} = 0.755$ (Table 5 data). Consider comparing rheological property estimates using procedure (10) and the log-linear method Andaverde et al. [3]. Table 4 shows the results of measurements on a FANN rotational viscometer with $\alpha = 0.9365$. Estimates of rheological properties $\hat{a}$ are constructed using the first procedure (10) in a similar way and are given in Table 5. According to the second procedure (10), the most adequate are the rheological models of Ostwald $\hat{\nu} = 2$ (Series KE-11) and Herschel–Balkley $\hat{\nu} = 3$ (Series KE-16) [24, 25]. Statistical characteristics of the rheological properties estimates:

σ_{τ_0}, Pa – / 0.5626;

σ_k, $Pa \cdot s^n$ 0.0212 / 0.0583;

σ_n 0.0182 / 0.0733;

$r_{\tau_0 k}$ – /-0.8562;

$r_{\tau_0 n}$ – / 0.8245;

r_{kn} -0.9938 / -0.9969.

Here, the numerator shows the performance values for Series KE-11 data, and the denominator shows Series KE-16 data.

The calculated values of shear stresses $\hat{\tau}_a$ for the most adequate rheological models and models (4), $\hat{\tau}_{LL}$ are constructed according to equations (5) and (6) and are shown in Table 5. The parameters of the rheological model (Andaverde et al. 2019) [3]: $\hat{a} = 22.524 \cdot 10^{-3}$Pa·s, $\hat{b} = -1.041 \cdot 10^{-5}$Pa·s^2, $\hat{c} = 2.83927 \cdot 10^{-3}$s, $\hat{\tau}_0 = 0.465797$ Pa (Series KE-11); $\hat{a} = 38.5731 \cdot 10^{-3}$Pa·s, $\hat{b} = 2.493 \cdot 10^{-5}$ Pa·s^2, $\hat{c} = 5.63793 \cdot 10^{-3}$ s, $\hat{\tau}_0 = 17.2358$ Pa (Series KE-16).

To compare estimates of rheological properties using procedure (10) and methods Bui and Tutuncu [16], Andaverde et al. [3], Tables 2 and 4 show the values of relative errors for shear stresses (δ_a, δ_{cs}, δ_{GB}, δ_{LL}), as well as integral indicators (sum of squared deviations $\sum_i (\tau_i - \hat{\tau}_i)^2$ and adequacy variance $\hat{\sigma}_a^2$). The indicator of the sum of the squared deviations is used in Table 1 to compare the processing results with the rheological model in the form of cubic splines (3), for which the number of parameters exceeds the number of measurements N.

The results of Tables 2 and 4 indicate the best integral indicators of the procedure (10) in comparison with the methods (Bui and Tutuncu [16], Andaverde et al. [3]), which are based on the shear rate gradient formula for a viscous fluid. It should be noted that for the rheological models adopted in the class ϑ, the number of parameters is less than for the models (3) or (4). For the spline rheological model (3), it is obvious that there are deviations between the calculated and measured values of shear stresses, which are caused by the use of (3) in the basic equation of rotational viscometry (5) and (6).

Figures 2 and 3 show rheograms of fluids (tables 2 and 4), as well as their rheological curves. It should be emphasized the distinctive deviation of rheograms for models (3) and (4) from the measurement data, the curves are located higher (Fig. 2 *a*, *c* and 3 *a*, *c*, as well as Tables 1 and 3). This has been highlighted in [21].

In quantitative terms, the deviations between the rheograms of fluids constructed using the methods (9), (10) and based on the shear rate gradient formula for a viscous fluid that depend on the relative viscometer clearance α and rheological properties a_ν. So, for example, for liquids (Fig. 2 *c*, *d* and 3 *c*, *d*), these deviations are not significant. The condition is obvious, the closer the rheological properties to the Newtonian ones, the smaller the deviations between rheograms (for example, for Ostwald, Herschel–Balkley and Schulman–Casson fluids at $n \to 1$ and $\tau_0 \to 0$ we have $\dot{\gamma} \to 2\omega / (1 - \alpha^2)$).

Biviscosity fluid

An analysis of the results of studies of the rheological properties of drilling fluids and cement slurries shows [14, 19, 23] that, in some cases, biviscosity models (1) most adequately describe the data of rotational viscometry.

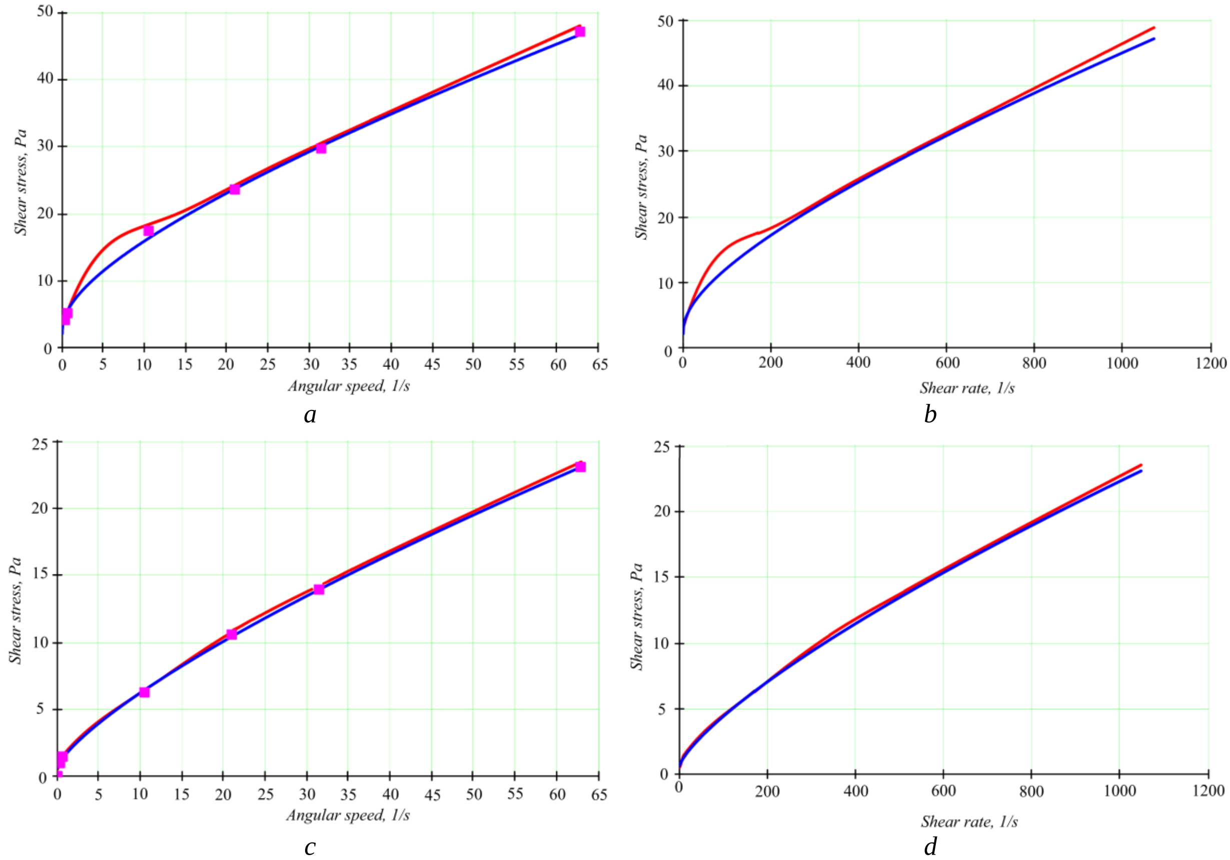
As an example, we consider the results of the interpretation of rotational viscometry data for a cement system based on Portland cement PCT I-50 with the addition of high-strength microspheres in the ratio 94 : 6 (water-cement ratio 0.6, thickening time 60 min). Measurement results on a rotational viscometer ($\alpha = 0.9365$) at 20°C: rotation speed, min^{-1} – 2, 50, 100, 200, 300, 400, 500, 600, 700; shear stress, Pa – 4.06, 10.15, 12.69, 16.25, 18.79, 22.85, 27.42, 32.49, 37.57.

Data processing was performed using procedure (10). The class ϑ included the rheological models of Bingham ($\nu = 1$), Ostwald ($\nu = 2$), Herschel – Balkley ($\nu = 3$), Shulman – Casson ($\nu = 4$) and biviscosity, represented by various combinations of Newton, Bingham and Ostwald ($\nu = 5 - 12$) models. Table 6 provides information on the adequacy variances of the rheological models of the cementing system PCT I-50. The most appropriate are the biviscosity models of Ostwald and Bingham ($\sigma_a^2 = 0.0814$ Pa^2), Ostwald and Ostwald ($\sigma_a^2 = 0.0945$ Pa^2), Ostwald and Newton ($\sigma_a^2 = 0.4667$ Pa^2). For two ($\nu = 1, 2$) and three parameter ($\nu = 3, 4$) models, the estimates of adequacy variance are much larger, and some biviscous models do not describe the experimental data (due to conditions (11) or meaningless parameter estimates $\tau_0 < 0$).

Table 6 – Estimates of the adequacy variance of rheological models of cement slurry PCT I-50

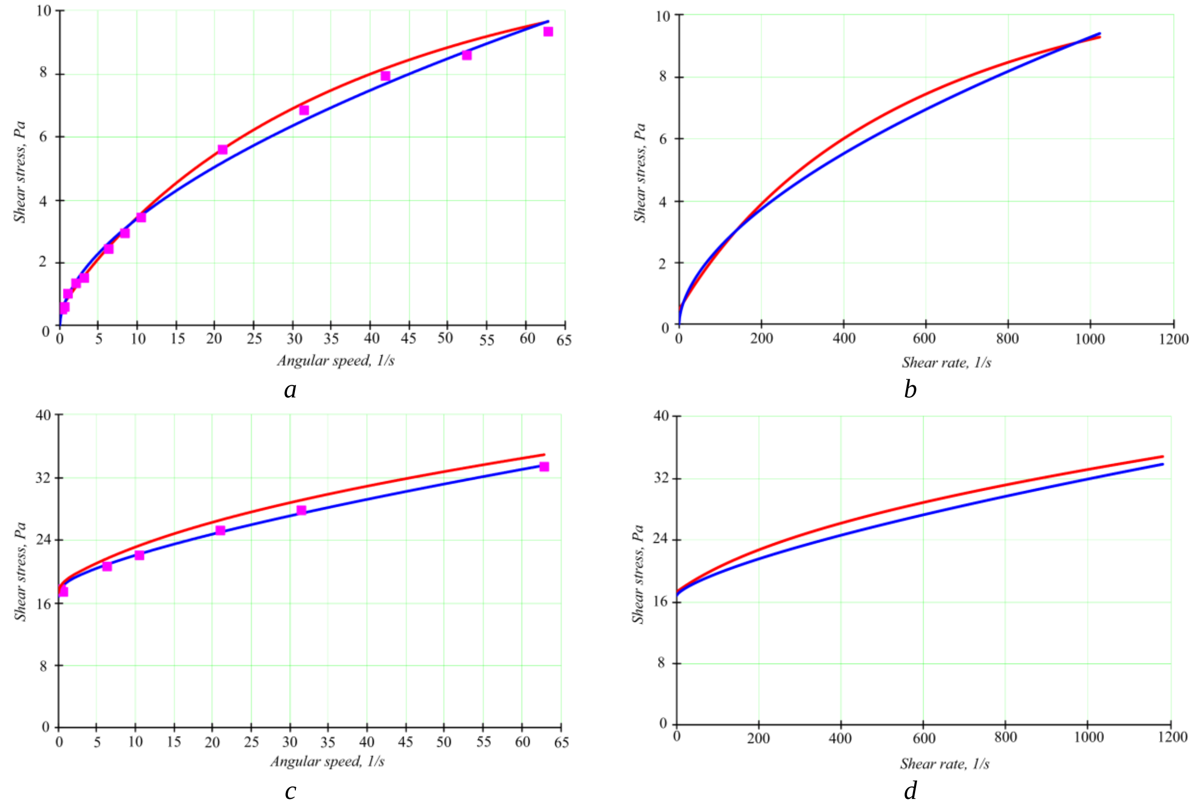
Rheological model ν	Variance σ_{av}^2, Pa^2
Bingham $\nu = 1$	2.1631
Ostwald $\nu = 2$	5.5490
Herschel–Balkley $\nu = 3$	2.5100
Shulman–Kesson $\nu = 4$	2.1534
Biviscous Newton and Bingham $\nu = 5$	∞ *
Biviscous Newton and Ostwald $\nu = 6$	∞
Biviscous Bingham and Newton $\nu = 7$	∞
Biviscous Bingham and Bingham $\nu = 8$	∞
Biviscous Bingham and Ostwald $\nu = 9$	∞
Biviscous Ostwald and Newton $\nu = 10$	0.4667
Biviscous Ostwald and Bingham $\nu = 11$	0.0814
Biviscous Ostwald and Ostwald $\nu = 12$	0.0945

Note*. Rotational viscometry data are not described by the rheological model.



a, *b* – Table 3 data (Bui and Tutuncu); *c*, *d* – Table 3 data (Bui and Tutuncu): most appropriate rheological model (*a*, *b* – Shulman – Caisson; *c*, *d* – Herschel – Balkley); cubic splines model; measurement data

Figure 2 – Rheograms (*a*, *c*) and rheological curves (*b*, *d*) of fluids



a, *b* – Series KE-11; *c*, *d* – Series KE-16; most appropriate rheological model (*a*, *b* – Ostwald; *c*, *d* – Herschel–Balkley); log-linear model; measurement data

Figure 3 – Rheograms (*a*, *c*) and rheological curves (*b*, *d*) of fluids

Table 7 – Assessment of the rheological properties of PCT I-50 cementing system

Rheological model $\hat{\nu}$	Rheological properties $\hat{a}_\nu$	Statistical characteristics
Biviscous Ostwald and Bingham $\hat{\nu}=11$	$\hat{k}^{(1)}=2.337$ Pa·s^n $\hat{n}^{(1)}=0.3254$ $\eta^{(2)}=0.0289$ Pa·s $\hat{\tau}_0^{(2)}=2.807$ Pa	$\sigma_{k^{(1)}}=6.15\cdot10^{-4}$ Pa·s$^{n^{(1)}}$, $\delta_{k^{(1)}}=0.2$ % $\sigma_{n^{(1)}}=1.70\cdot10^{-3}$, $\delta_{n^{(1)}}=0.5$ %, $r_{kn^{(1)}}=-0.8862$ $\sigma_{\eta^{(2)}}=5.30\cdot10^{-4}$ Pa·s, $\delta_{n^{(2)}}=1.8$ % $\sigma_{\tau_0^{(2)}}$=0.4394 Pa, $\delta_{\tau_0^{(2)}}$=15.7%, $r_{\tau_0\eta^{(2)}}=-0.9623$
Biviscous Ostwald and Ostwald $\hat{\nu}=12$	$\hat{k}^{(1)}=2.337$ Pa·s^n $\hat{n}^{(1)}=0.3254$ $\hat{k}^{(2)}=0.0635$ Pa·s^n $\hat{n}^{(2)}=0.8996$	$\sigma_{k^{(1)}}=0.2684$ Pa·s$^{n^{(1)}}$, $\delta_{k^{(1)}}=11.5$ % $\sigma_{n^{(1)}}=0.0159$, $\delta_{n^{(1)}}=4.9$ %, $r_{kn^{(1)}}=-0.9935$ $\sigma_{k^{(2)}}=1.08\cdot10^{-3}$ Pa·s^n, $\delta_{k^{(2)}}=1.7$% $\sigma_{n^{(2)}}=1.86\cdot10^{-3}$, $\delta_{n^{(2)}}=0.2$ %, $r_{kn^{(2)}}=-0.9591$
Biviscous Ostwald and Newton $\hat{\nu}=10$	$\hat{k}^{(1)}=1.894$ Pa·s^n $\hat{n}^{(1)}=0.3677$ $\hat{\eta}^{(2)}=0.0318$ Pa·s	$\sigma_{k^{(1)}}=0.4341$ Pa·s$^{n^{(1)}}$, $\delta_{k^{(1)}}$=22.9 % $\sigma_{n^{(1)}}=0.030$, $\delta_{n^{(1)}}=8.15$ %, $r_{kn^{(1)}}=-0.9953$ $\sigma_\eta^{(2)}=7.66\cdot10^{-4}$ Pa·s, $\delta_{\eta^{(2)}}=2.4$ %

Table 7 shows the rheological properties and their statistical characteristics for three most appropriate biviscosity models. Figure 4 shows the rheograms of these models, in which, in accordance with (7), three distinctive flow regions in the viscometer gap are identified.

Informative content of experiments

The task of rotational viscometry data processing (8), algorithms for solving it (9) and (10) include information about content of the experiments, i.e. covariance matrix C of random component. In principle, the matrix C may be unknown (data processing is carried out according to single measurements) or known from the experiment.

Let us consider the influence of the form of the covariance matrix on the results of processing the rheological properties measurement data of a biopolymer drilling fluid with a density 1300 kg/m^3 (16 Luitnyaska well with a measured depth of 2463 m). Ten samples of drilling fluid were taken and at a temperature of 20 °C measurements were taken on a viscometer FANN 800 (α=0.9365): rotation frequency – 3, 6, 30, 60, 100, 200, 300, 600 min^{-1}; average of shear stresses – 4.788, 5.985, 11.49, 16.28, 20.59, 29.69, 36.87, 53.63 Pa. On the basis of the results of the measured shear stresses, a covariance matrix of the random component is built (16)

Tables 8 and 9 show the data processing results for various covariance matrices:stationary uncorrelated with an unknown variance of adequacy (it is assumed that the average values of shear stresses were obtained from measurements on one sample), unsteady uncorrelated (the matrix is represented by elements of the main diagonal), and unsteady correlated (matrix C is represented by all elements).

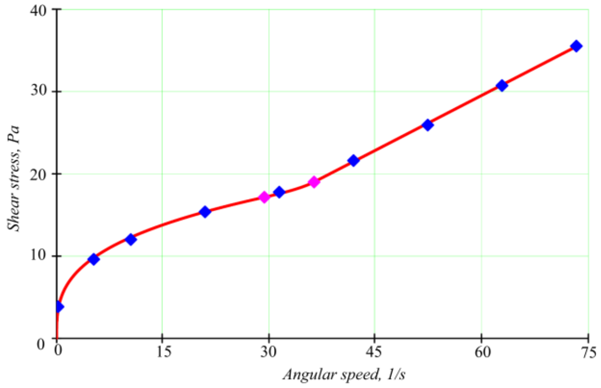
The results of viscometry data processing indicate a significant effect of the information content of the experiments on the estimates of rheological properties and are consistent with previous studies [19]. In some cases, this may lead to the choice of a different rheological model (Table 8). It should also be noted the influence of the type of matrix C on the accuracy of the assessment of rheological properties (Table 9).

Multicriterial interpretation

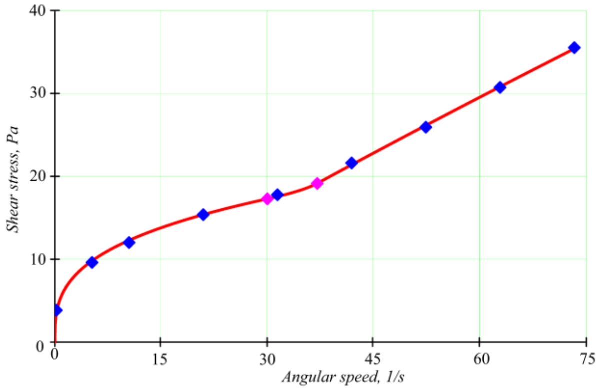
Rheological model

In the applied aspect, estimates of the rheological model $\hat{\nu}$ and properties $\hat{a}_\nu$ are close lyrelated to making certain technological decisions, the features of which impose certain requirements on information support. In such cases, it is advisable to evaluate the model $\hat{\nu}$ and properties $\hat{a}_\nu$ of the studied fluid on the basis of a multicriteria analysis of the data processing results.

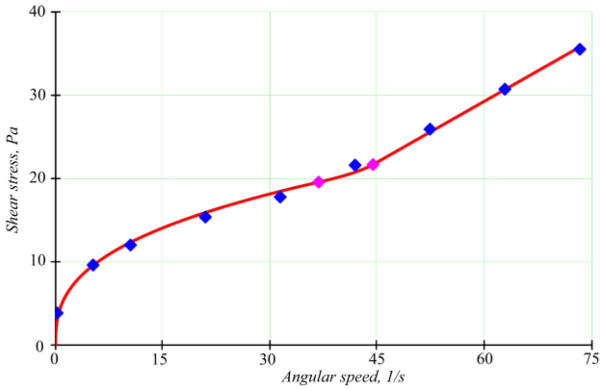
$$C=\begin{vmatrix} 0.086605 & 0.035661 & 0.112078 & 0.156654 & 0.173211 & 0.266184 & 0.287835 & 0.494160 \\ 0.035661 & 0.022925 & 0.048397 & 0.080237 & 0.084058 & 0.132455 & 0.135002 & 0.213966 \\ 0.112078 & 0.048397 & 0.153470 & 0.210782 & 0.227339 & 0.358521 & 0.386540 & 0.653361 \\ 0.156654 & 0.080237 & 0.210782 & 0.319039 & 0.341964 & 0.536826 & 0.564845 & 0.933555 \\ 0.173211 & 0.084058 & 0.227339 & 0.341964 & 0.375078 & 0.580128 & 0.617063 & 1.020160 \\ 0.266184 & 0.132455 & 0.358521 & 0.536826 & 0.580128 & 0.909993 & 0.962211 & 1.590740 \\ 0.287835 & 0.135002 & 0.386540 & 0.564845 & 0.617063 & 0.962211 & 1.050090 & 1.707910 \\ 0.494160 & 0.213966 & 0.653361 & 0.933555 & 1.020160 & 1.590740 & 1.707910 & 2.914020 \end{vmatrix}. \quad (16)$$



a



b



c

a – Ostwald and Bingham;
b – Ostwald and Ostwald;
c – Ostwald and Newton;
—— rheological model;
◆ ◆ measurement data;
◆ ◆ points τ^* and τ^*/α^2

Figure 4 – Rheograms of biviscosity models of the cementing system based on portland cement PCT I-50 with the addition of high-strength microspheres

The analysis of the rotational viscometry data processing results demonstrates that for a sufficiently complete class ϑ of rheological models, as a rule, several models with similar best values correspondt othe maximum likelihood function criterion. Then, other criteria can beused to select a rheological model, for example, based onan analysis of covariance matrixes O^{ν} of rheological property estimates [26, 27]:

$$k_1(\hat{a}_\nu) = \left(\prod_j \sigma_{\nu j}/\hat{a}_{\nu j}\right)^{1/r_\nu}, \qquad (17)$$

$$k_2(\hat{a}_\nu) = \frac{1}{r_\nu}\sum_j \sigma_{\nu j}/\hat{a}_{\nu j}, \qquad (18)$$

$$k_3(\hat{a}_\nu) = \max\left(\sigma_{\nu j}/\hat{a}_{\nu j}\right), \qquad (19)$$

where $\sigma_{\nu j}$ is estimation of the standard deviation j – th index of the ν – th rheological model; r_ν is number of rheological properties.

Criteria (17) and (18) determine the geometric mean and arithmetic mean, respectively, and (19) determines the maximum values of the relative standard deviations of the estimates of rheological properties.Similar estimates for individual indicators of rheological properties can also be used as criteria.

Processing of rotational viscometry data using multicriteria assessments narrows down to implementation of the following procedures:

to form a class ϑ of possible rheological models (including biviscosity);

to identify the most adequate using procedures (9) or (10) ν_* rheological model;

to high light a subset $\vartheta_e \in \vartheta$ of equivalent to ν_* rheological models;

to develop matrices of covariance estimates of rheological properties for equivalent models

$$O^{\nu} = \left(A'^{*}(\omega,\hat{a}_\nu)C^{-1}A'(\omega,\hat{a}_\nu)\right)^{-1}, \ \nu \in \vartheta_e;$$

to select a rheological model $\hat{\nu}$ on the basis of the analysis of covariance matrixes O^{ν} of property estimates

$$\min k(\hat{a}_\nu) \Rightarrow \hat{\nu}, \ \nu \in \vartheta_e.$$

A subset ϑ_e is highlighted, forexample, by checking the sequence of statistical hypotheses or expert methods. In the particular case, when ϑ_e is represented by one element, we have the choice of the rheological model in accordance with the principle of the maximum likelihood function (9) or (10).

Analysis of covariance matrices O^{ν} of a subset of equivalent models includes substantiating a criterion or criteria $k(\hat{a}_\nu)$, constructing their estimates, and choosing a rheological model $\hat{\nu}$.

It should be noted that the described procedures for the interpretation of rotational viscometry data are valid for conventional and batch processing of measurement results. In the latter case, criteria (13) or (14) and generalizations (17)–(19) are applied to the experimental design.

We illustrate the elements of the multicriteria analysis of the cementing slurry system data based on portland cement PCT I-50 with the addition of high-strength microspheres (Section 3.2). A subset of equivalent models ϑ_e for $\hat{\nu}_* = 11$ includes the biviscosity fluids of Ostwald and Ostwald $\hat{\nu} = 12$ and Ostwald and Newton $\hat{\nu} = 10$ (Table 5 and Figure 3). This can be seen by checking the totality of statistical

Table 8 – Criteria for choosing a rheological model depending on the type of covariance matrix

Model v	Covariance matrix C		
	stationary uncorrelated	non-stationary uncorrelated	non-stationary correlated
Bingham, 1	14.07	236.7	76120
Ostwald, 2	0.6422	17.7	4635
Herschel – Bulkley, 3	0.0135	0.177	27.15
Schulman – Casson, 4	0.0345	0.3538	4.927

Note. For the stationary uncorrelated matrix C the values of the adequacy variance (Pa^2) are given, and for the other matrices C, the transformed residual.

Table 9 – Estimates of rheological properties depending on the type of covariance matrix

Matrix C	Model $\hat{v}$	Rheological properties $\hat{a}_v$	Statistical characteristics
Stationary uncorrelated	3	$\hat{\tau}_0 = 2.186$ Pa $\hat{k} = 0.8532$ Pa$\cdot$s^n $\hat{n} = 0.5815$	$\sigma_{\tau_0} = 0.1193$ Pa , $\sigma_k = 0.0255$ Pa$\cdot$s^n $\sigma_n = 0.0041$, $r_{\tau_0 k} = -0.8868$ $r_{\tau_0 n} = 0.8531$, $r_{kn} = -0.9958$
Non-stationary uncorrelated	3	$\hat{\tau}_0 = 2.049$ Pa $\hat{k} = 0.9007$ Pa$\cdot$s^n $\hat{n} = 0.5734$	$\sigma_{\tau_0} = 0.3859$ Pa , $\sigma_k = 0.1431$ Pa$\cdot$s^n $\sigma_n = 0.0243$ $r_{\tau_0 k} = -0.9441$, $r_{\tau_0 n} = 0.9189$ $r_{kn} = -0.9928$
Non-stationary correlated	4	$\hat{\tau}_0 = 0.3624$ Pa $\hat{\eta} = 0.0024$ Pa$\cdot$s $\hat{n} = 5.593$	$\sigma_{\tau_0} = 0.0192$ Pa , $\sigma_\eta = 0.0001$ Pa$\cdot$s $\sigma_n = 0.0702$ $r_{\tau_0 \eta} = 0.7927$, $r_{\tau_0 n} = -0.943$ $r_{\eta n} = -0.9494$

hypotheses about the equality of variances of adequacy $H_{0s}: \ \sigma_{a11}^2 = \sigma_{as}^2, \ s = \{10,12\}$.

Table 10 for a subset of equivalent models provides estimates of the selection criteria for the rheological model (10) and (17)–(19). The latter are calculated according to the data in Table 7. The rheological model of Ostwald and Bingham and the (19) Ostwald and Ostwald best correspond to the accuracy indicators of the rheological properties estimates (17) and (18).

State equation

In decision-making tasks, models are needed that describe the dependences of the properties (including rheological) of drilling fluids on state parameters (temperature, pressure, concentration of reagents, etc.). Note the original model for estimating shear stresses at various rotational speeds of a rotational viscometer versus pressure and temperature [7], which subsequently allows one to obtain these dependences for rheological properties. The construction of such models or state equations is associated with the use of experimental design methods.

In case of a significant number of factors and their levels of change, different experimental designs are used (Latin, Greek–Latin and others), the main advantage of which is a complete range of factors with a minimum number of experiments. To process such experiments, it couldbe of interest to use the methods of regression analysis in the class of polynomial basis functions [15]

$$a_{vj}(x) = B^\varepsilon b^\varepsilon(x), \ \varepsilon \in \vartheta_e, \tag{20}$$

where $b^\varepsilon(x)$, B^ε are vectors of basis functions and parameters ε – th model. For example, a vector $b^\varepsilon(x)$ is formed on the basis of the linear part of polynomials and includes arbitrary combinations of products and squares of variable factors. The total number of parameters in the class E of basis functions should be less than the volume M of the experimental design.

The multi-criterion procedure for selecting the equations of state (20) of rheological properties is implemented in the following sequence:

to form a class E of possible equations of state for basis functions $b^\varepsilon(x)$;

to build from the condition

$$\min\left\{\sigma_{a\varepsilon}^2 = \frac{1}{N - r_\varepsilon}\sum_{i=1}^{N}\left[B_j^\varepsilon b^\varepsilon(x_i) - a_{vj}(x_i)\right]^2\right\} \Rightarrow \left\{\hat{B}_j^\varepsilon, \varepsilon_*\right\};$$

to estimate parameters $\hat{B}_j^\varepsilon$ and select the most adequate ε_* equation of state for the j – th indicator of rheological properties;

to select a subset E_e equivalent for ε_* equations of state;

to construct covariance matrices O^{ε} for parameters estimates of subset E_e of equivalent equations of state;

to select state equations $\hat{\varepsilon}$ based on the analysis of covariance matrix O^{ε} estimates of their parameters.

Note that in this case, the multi-criterion analysis procedure of equations of state in the applied aspect has more possibilities. This is due to the formation of basis functions in (20), the power of the subset E_e, the criterial requirements for estimating the parameters of equations, etc.In particular, for the analysis of equations of state, approaches based on the ideas of ranking criteria and the sequential narrowing of subsets E_e can be used by extracting the most effective equations out of them.

Consider an example of constructing the equation of state of the rheological properties of a cement slurry based on cement PCT I-100 (water–cement ratio 0.5; density 1820 kg/m^3) [27]. After mixing, the cement slurry was conditioned with an atmospheric consistometer to a predetermined temperature. Using a OFITE 800 ($\alpha = 0.9365$) rotational viscometer, we studied the effect of time from the beginning of mixing $t = (10,\ 20, 30, 40, 50)$ min, temperature $T = (22,\ 30, 40, 50, 60)$ °C and rotation frequency $\Omega = (75,\ 100, 125, 150, 175)$ min^{-1} of atmospheric consistometer on the rheological properties of cement slurry.

Table 11 shows the experimental design, as well asusing procedures (13) and (14), the results of rheological properties assessments in the class ϑ of rheological models Bingham ($\nu = 1$), Ostwald ($\nu = 2$), Herschel–Balkley ($\nu = 3$) and Shulman–Casson ($\nu = 4$):

Herschel–Balkley ($\nu = 3$) $\hat{\sigma}_3^2 = 0.4148$ Pa^2;

Bingham ($\nu = 1$) $\hat{\sigma}_1^2 = 1.2590$ Pa^2;

Shulman–Casson ($\nu = 4$) $\hat{\sigma}_4^2 = 1.8310$ Pa^2;

Ostwald ($\nu = 2$) $\hat{\sigma}_2^2 = 4.1990$ Pa^2.

The Herschel–Balkley rheological model is most appropriate for the experiment plan/layout ($\hat{\nu} = 3$).

The state equations for the rheological properties of the most adequate ε_* Herschel–Balkley model (Table 10) are built in the form

$$a_{\nu i}(x) = \exp\left[B_i^{\varepsilon} b_i(x)\right], \varepsilon \in E, \qquad (21)$$

similar to equation (20) after its linearization. The class of models E is formed from the linear part of second-order polynomials taking in to account arbitrary combinations of multiplication and squares of variable factors (Table 12) and includes 51 models. The units of measurement of model parameters (19) correspond to the units of measurement of rheological properties and variable factors (Table 11).

Table 13 shows the estimates of the most adequate equations of state for the indicators of rheological properties according to the criterion of adequacy variance for the linearized form of equations (21), subsets E_e of equivalent equations are selected, and their estimates are constructed according to the criteria (17)–(19) of covariance matrices. Note that the criteria $k_{lp}\left(B^{\varepsilon}\right)$ and $k_{np}\left(B^{\varepsilon}\right)$ meet the criterion (18) for linear and nonlinear parameters of the state equations, respectively. Table 13 highlights the state equations based on the results of multicriteria analysis. The parameters of these equations of state are given in Table 12.

Figure 5 shows the time dependences of the rheological properties of the PCT I-100 cementing slurry at a temperature of 40 °C and various values of the rotational frequency Ω of the atmospheric consistometer. These dependencies are constructed for the equations of state highlighted in Table 13.

Conclusions

Using the principle of maximum likelihood function, a model of rotational viscometry data processing (Single processing) is considered, which is based on a rigorous solution of the basic equation of rotational viscometry, takes into account the information content of the experiments, estimates the most appropriate rheological model and properties of the liquid, as well as their statistical characteristics. The class of rheologically stationary models allows an explicit analytical representation of the form $\dot{\gamma} = \dot{\gamma}(\tau)$ and includes biviscosity fluids.

The model for rotational viscometry data processing for a data array (Batch processing) was generalized, the feature of which is the choice of the most appropriate rheological model for the experimental design. The data processing model is valuable for constructing the equations of state of rheological properties and is implemented for cases where the covariance matrix of the random component is unknown or known from experiment.

For the relative gaps of the OFITE 800 viscometer, the influence of the rheological properties of the Herschel–Balkley fluid on the accuracy of their estimates was studied using the techniques constructed using formula (15) for the shear rate gradient of a viscous fluid. It is shown that for large annular gaps, the errors in the estimates of rheological properties are high. Examples of rotational viscometry data processing of drilling fluids are presented, illustrating the interpretation advantages of the proposed models for rheological properties estimating.

The ideas of multi-criterion analysis in the interpretation of the processing results (including batch) of rotational viscometry data are proposed. This is achieved by using procedures for constructing a subset of equivalent models for the likelihood function criterion and selecting a rheological model based on criteria for covariance matrices of rheological property estimates. The possibility of using multi-criterion analysis to select the equations of state of rheological properties was considered.

Table 10 – Multicriteria evaluation of processing data results of rotational viscometry

Rheological model v	Criteria for choosing a rheological model $v \in \vartheta_e$			
	σ^2_{av}, Pa^2	$k_1(\hat{a}_v)$	$k_2(\hat{a}_v)$	$k_3(\hat{a}_v)$
Biviscous Ostwald and Bingham $v=11$	0.0814	0.0079	0.0451	0.1565
Biviscous Ostwald and Ostwald $v=12$	0.0945	0.0211	0.0457	0.1148
Biviscous Ostwald and Newton $v=10$	0.4667	0.0766	0.1116	0.2292

Table 11 – Experimental design and rheological property assessment results of PCT I-100 cement slurry

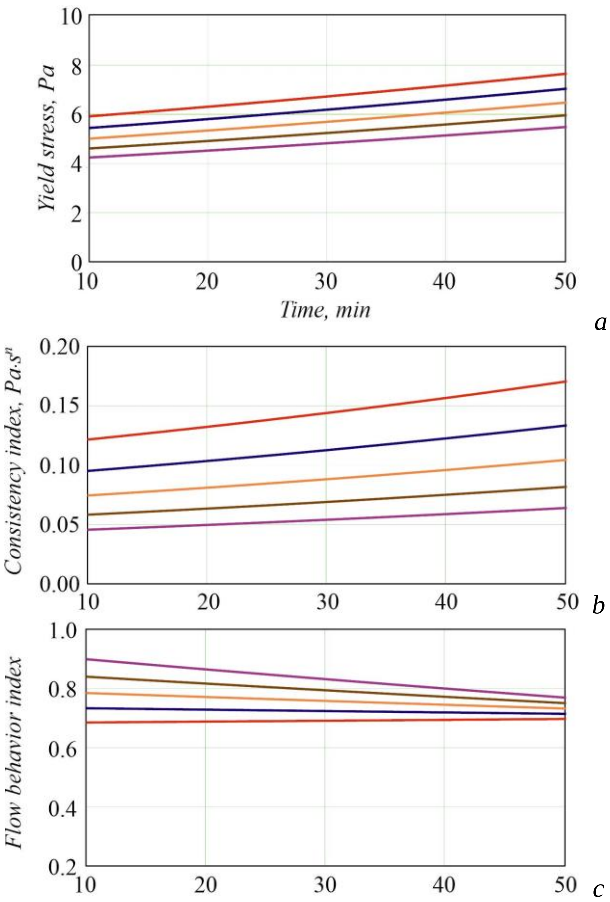
Test	Factors			Rheological properties		
	t, min	T, °C	Ω, min^{-1}	$\hat{\tau}_0$, Pa	$\hat{k}$, Pa·s^n	$\hat{n}$
1	10	22	150	5.382	0.1378	0.7550
2	10	30	75	4.144	0.0899	0.7810
3	10	40	125	4.133	0.0727	0.8120
4	10	50	175	3.881	0.0904	0.7530
5	10	60	100	3.128	0.0220	0.9576
6	20	22	175	7.579	0.2278	0.6795
7	20	30	150	6.356	0.1734	0.7014
8	20	40	75	4.579	0.0433	0.8840
9	20	50	100	3.619	0.0532	0.8441
10	20	60	125	6.656	0.1572	0.6817
11	30	22	125	6.634	0.0654	0.8447
12	30	30	100	6.010	0.0602	0.8798
13	30	40	150	7.393	0.1160	0.7492
14	30	50	75	5.261	0.0118	1.0670
15	30	60	175	5.440	0.1400	0.7130
16	40	22	100	3.720	0.0416	0.9327
17	40	30	125	6.030	0.4291	0.5817
18	40	40	175	6.019	0.1135	0.7658
19	40	50	150	4.793	0.0729	0.8025
20	40	60	75	3.289	0.0601	0.8233
21	50	22	75	6.178	0.1513	0.7482
22	50	30	175	10.090	0.2749	0.6700
23	50	40	100	7.187	0.1051	0.7750
24	50	50	125	6.185	0.6156	0.5307
25	50	60	150	2.454	0.0070	1.0940

Table 12 – Equations parameters state indicators of rheological properties of cement slurry PCT I-100

Basic functions b_i	Model parameters (18)	Model parameter B_i^ε values (18)		
		τ_o	k	n
1	B_1^ε	-0.42005	-2.91907	0.52526
t	B_2^ε	0.02274	0.00903	- 0.00716
T	B_3^ε	0.05607	-0.02434	-0.00159
Ω	$B_4^\varepsilon \cdot 10^3$	9.60842	9.44056	- 3.05233
tT	$B_5^\varepsilon \cdot 10^4$	-4.06831	–	–
$t\Omega$	B_6^ε	–	–	–
$T\Omega$	$B_7^\varepsilon \cdot 10^4$	-1.57379	–	–
$tT\Omega$	$B_8^\varepsilon \cdot 10^6$	–	–	1.06174
t^2	B_9^ε	–	–	–
T^2	$B_{10}^\varepsilon \cdot 10^4$	- 4.36133	–	1.69268
Ω^2	B_{11}^ε	–	–	–

Table 13 – Multicriteria analysis of equations state rheological properties of the cement system PCT I-100

Model $\varepsilon \in \vartheta_e$	Variance of adequacy	Parameters number	Covariance Matrix Criteria O^{ε}				
			$k_1(B^{\varepsilon})$	$k_2(B^{\varepsilon})$	$k_3(B^{\varepsilon})$	$k_{lp}(B^{\varepsilon})$	$k_{np}(B^{\varepsilon})$
Yield point							
33	0.06912	7	1.0180	1.2105	3.3415	1.5549	0.7512
19	0.06924	6	1.2647	9.2454	52.3558	1.2905	0.8749
34	0.06975	7	1.1136	1.2764	2.9989	1.5895	0.8590
25	0.07047	7	0.7717	0.8887	2.3113	0.9941	0.7482
18	0.07071	6	0.8706	1.1491	3.3819	1.3703	0.7067
Consistency index							
1	0.02040	4	0.5531	0.6768	1.4698	0.6768	0
7	0.02091	5	1.0186	1.1703	1.9647	1.1280	1.3395
6	0.02105	5	0.9487	1.5875	4.0161	1.3354	2.5959
8	0.02119	5	1.4582	1.8675	4.1916	1.2865	4.1916
4	0.02121	5	0.7660	2.7956	10.8480	3.4881	0.0257
5	0.02127	5	0.9172	1.7497	6.4075	1.9923	0.7794
Power Law index							
5	0.01605	5	1.0802	1.2424	2.4829	1.3104	0.9707
1	0.01620	4	1.1784	1.3265	2.2128	1.3265	0
19	0.01624	6	0.9093	0.9460	1.2864	0.8633	1.1113
18	0.01657	6	2.5933	13.6268	74.1930	19.7793	1.3217
7	0.01662	5	1.3586	1.5005	2.3594	1.5222	1.4137



a – yield stress; b – consistency index; c – power law index) on the time of the beginning of the mixing of PCT I-100 cement slurry at temperature 40 °C and rotation frequency Ω , min^{-1}:
75; 100; 125; 150; 175

Figure 5 – Dependence of the rheological properties

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Визначення реологічних властивостей бурових технологічних рідин за даними ротаційної віскозиметрії

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Розглянуто метод обробки даних ротаційної віскозиметрії, який ґрунтується на принципі максимуму функції правдоподібності, враховує інформаційну змістовність дослідів і побудований на строгому розв'язку основного рівняння течії Куетта в кільцевому зазорі віскозиметра. Клас моделей сформовано із реологічно стаціонарних (в тому числі бів'язких) рідин. Узагальнено моделі обробки даних плану експерименту з метою побудови рівнянь стану реологічних властивостей від змінних факторів. Запропоновано мультикритеріальну інтерпретацію оцінок реологічної моделі та властивостей рідин. Наведено ілюстративні приклади оцінок реологічних властивостей бурових технологічних рідин і побудови рівнянь їх стану.

Ключові слова: *бів'язка рідина; мультикритеріальний аналіз; принцип максимуму функції правдоподібності; реологічно стаціонарні моделі; рівняння стану; течія Куетта.*