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Adaptation of the material balance of a gas deposit

Yu. Zarubin, M. Hunda, P. Mamus*

Scientific Research «Institute of Oil and Gas Industry» of National Joint-Stock Company «Naftogaz of Ukraine»; Vyshneve, Kyiv region, Ukraine

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Abstract

The aim of the research is to study the application of the material balance method with modern technologies to create a useful gas reservoir model with water influx.

The basic equations of material balance are analyzed taking into account the Mendeleev–Clapeyron law for gases. The analysis was performed taking into account the water influx component according to Fetkovich equations. The paper highlights the problems of identifying the parameters of the material balance model as material balance model doesn't include the geological structure of deposits. The calculation was done by material balance models created on Mathcad and VBA Excel.

The analysis and examples given in the article testify to the expediency of applying the method of material balance with VBA Excel to clarify gas reserves, including those with a water-driven or mixed mode of development and obtain an adequate model of the reservoir.

Keywords: *deposits, engineering solutions, field, gas, material balance equation, water influx.*

The main equation of the material balance of a gas reservoir

The basic equation of the material balance of a gas reservoir is formulated on the basis that the sum of the masses of residual gas in the reservoir and the mass of gas produced is equal to the mass of the initial gas reserves. When using the generalized Mendeleev–Clapeyron law as the equation of state for a real gas, the material balance at time *t* is written in the following form

$$\frac{p(t)}{z\left[p(t),T_{in}\right]T_{in}}Vpor(t)= \\ =\frac{p_{st}}{z\left[p_{st},T_{st}\right]T_{st}}\left[Rzv_{ini}-Vprod(t)\right], \qquad (1)$$

were p_{st}, T_{st} pressure and temperature for standard conditions of production gas metering; T_{in} is reservoir temperature; $p(t)$ is current reservoir pressure; Rzv_{ini} is the volume of the initial drained reserves, reduced to standard conditions; $Vprod(t)$ is the accumulated volume of produced gas; $z\left[p(t),T_{in}\right]$ is the gas compressibility factor under the appropriate conditions; $Vpor(t)$ is the current pore volume occupied by gas in the reservoir.

* Corresponding author: zarubinua@gmail.com

There is a number of reasons leading to a change in the drained pore volume. Many of these reasons, such as the compressibility of rocks, the compressibility of bound water, retrograde phenomena, can be neglected without losing the accuracy of the material balance. It is not difficult to estimate that the compressibility of rocks can change the volume of void space by tenths of a percent, and retrograde phenomena of gas condensate systems by thousandths of a percent. It is unacceptable to ignore the change in the drained pore volume as a result of the influx of aquifer water into the productive area, connection, or disconnection of individual layers. An overview of modern correlations used in the equations of material balance can be found in the paper [4].

The use of the Fetkovich [1] method, which is an implementation of the principle of sequential change of stationary states, makes it possible, without significant loss of accuracy, get away to use an algorithmically inconvenient superposition method and go to a simple recurrent scheme for calculating water volumes at the following time step:

$$W_e(t_i)=W_e(t_{i-1})+\Delta W_e(\Delta t). \qquad (2)$$

Then the volume of pores occupied by gas at each moment of time decreases by the volume of water introduced into the reservoir:

$$Vpor(t)=Vpor(t)-W_e(t_i). \qquad (3)$$

To calculate the volumes of water influx $W_e(t)$ at the time *t* at constant pressure on the inner boundary of the aquifer (gas-bearing contour), Fetkovich obtained the formula:

$$W_e(t)=\frac{W_e}{p_{in}}\left[p_{in}-p\right]\left[1-\exp\left(-\frac{j_w p_{in}}{W_e}t\right)\right], \qquad (4)$$

where p_{in} is the initial reservoir pressure in the aquifer, which can be considered equal to the initial reservoir pressure in the reservoir, reduced to the position of the gas contour; p is the reservoir pressure on the gas content contour, which, with small assumptions, can be taken equal to the weighted average pressure in the gas part of the reservoir, used in the main material balance equation (1); W_e is potential and j_w is productivity index of the aquifer, respectively.

To get away from the principle of superposition, the use of which is necessary at variable pressure on the inner contour of the aquifer, and the possibility of using a recurrent calculation scheme, Fetkovich proposed a formula that allows calculating the volumes of penetrated water at intervals for each time step:

$$\Delta W_e(\Delta t) = \frac{W_e}{p_{in}}\left[p_w\left(t-\Delta t\right)-\overline{p}\left(t\right)\right]\times \\ \times\left[1-\exp\left(-\frac{j_w p_{in}}{W_e}\Delta t\right)\right], \quad (5)$$

where $p_w\left(t-\Delta t\right)$ is the average pressure in the aquifer not in the previous time step amd it is determined by the material balance between the initial elastic water reserves in it W_e and the total amount of water influx the productive part for the entire development period of the object W_{e_t} :

$$p_w\left(t\right) = p_{in}\left[1-\frac{W_{e_t}}{W_e}\right]; \quad (6)$$

$\overline{p}\left(t\right)$ is the average pressure on the internal circuit in the time interval Δt:

$$\overline{p}\left(t\right) = \frac{p\left(t\right)+p\left(t-\Delta t\right)}{2}. \quad (7)$$

As the testing of algorithms has shown, the use of the current pressure $p\left(t\right)$ instead of the average pressure on the internal circuit slightly increases the accuracy of the calculations at the initial sections of the time scale.

Thus, formally, the model of the material balance of a gas reservoir is a function $p_{pl}\left(t_i, \mathrm{R}\,zv_{ini}, p_{ini}, W_e, j_w\right)$, which determines the weighted average reservoir pressure in which the time t_i acts as an independent variable, and the parameters (coefficients) of the initial reservoir pressure are p_{ini}, initial gas reserves are $\mathrm{R}\,zv_{ini}$, W_e is potential and j_w is productivity index of the aquifer, respectively.

The transcendental nature of the material balance equation (1) excludes an analytical way of solving it and involves the use of numerical methods. Formally, it is necessary to find the pressure and volume of the embedded water that satisfy this equation and the equation of water inflow. Considering that the starting points for all variables are known, the most effective is the use of a simple, however, absolutely convergent bisection method for solving nonlinear equations.

Meanwhile, it is possible to construct several design schemes. Firstly, the application of the Fetkovich method excludes the possibility of constructing a completely explicit-implicit algorithm, due to the need to calculate the volume of the invading water at specified intervals. Therefore, the material balance algorithm for calculating the dynamics of reservoir pressure has an explicit recursive character in time. When solving the system of balance equations at each time layer, it is possible to apply the iterative search for the solution of the system of equations in the form according to an explicit scheme with simultaneous determination of the current pressure and volume of invading water and to use the absolutely stable method of running the calculation, when the reservoir pressure is calculated in accordance with the volume of invading water at the previous moment of time, and the volume of water at the current time layer according to the already determined pressure. This is a completely implicit calculation scheme.

Both algorithms are implemented in Visual Basic for Application (VBA) for the Excel program of the Microsoft Office application suite. A screenshot of the worksheet for calculating the main indicators of gas reservoir development is shown in Figure 1.

Testing the software module of material balance equations

Testing of the developed software module of material balance equations (MBE) was carried out by comparing the calculated dynamics of reservoir pressure and the volume of invading water obtained by different methods.

As a standard, there is used an object that simulates an idealized closed radially symmetric circular reservoir with a circular aquifer (Table 1), which allows using the exact solution obtained by Van Evendinger and W. Hurst without additional assumptions (A. F. Van Everdingen, W. Hurst, 1949) [2].

For a variable pressure at the inner boundary, based on the principle of superposition, the volume of water introduced into the reservoir is represented as the Duhamel`s integral:

$$W_{e_t} = \frac{2\pi kh}{\mu}\int_0^t \frac{\partial p\left(t\right)}{\partial t} Q\left(t-\tau\right)d\tau. \quad (8)$$

Here $Q\left(t-\tau\right)$ is the kernel of the Duhamel`s integral is the solution of Van Evendinger and V. Hars for constant pressure on the circuit between the productive and aquifer areas, which can be represented as:

$$Q\left(Fo\right) = \frac{R_a^2-1}{2} - \\ -2\sum_{i=1}^{\infty} e^{-\alpha_n^2 Fo}\frac{I_1^2\left(\alpha_n R_a\right)}{\alpha_n^2\left[I_0^2\left(\alpha_n\right)-I_1^2\left(\alpha_n R_a\right)\right]}, \quad (9)$$

where $R_a = \frac{r_a}{r_g}$ is dimensionless radius – the ratio of the outer radius of the aquifer r_a to the inner r_g (radius of the gas contour); $Fo = \frac{\gamma}{r_g^2}t$ is dimensionless time (Fourier criterion); r_g is characteristic size, in this case,

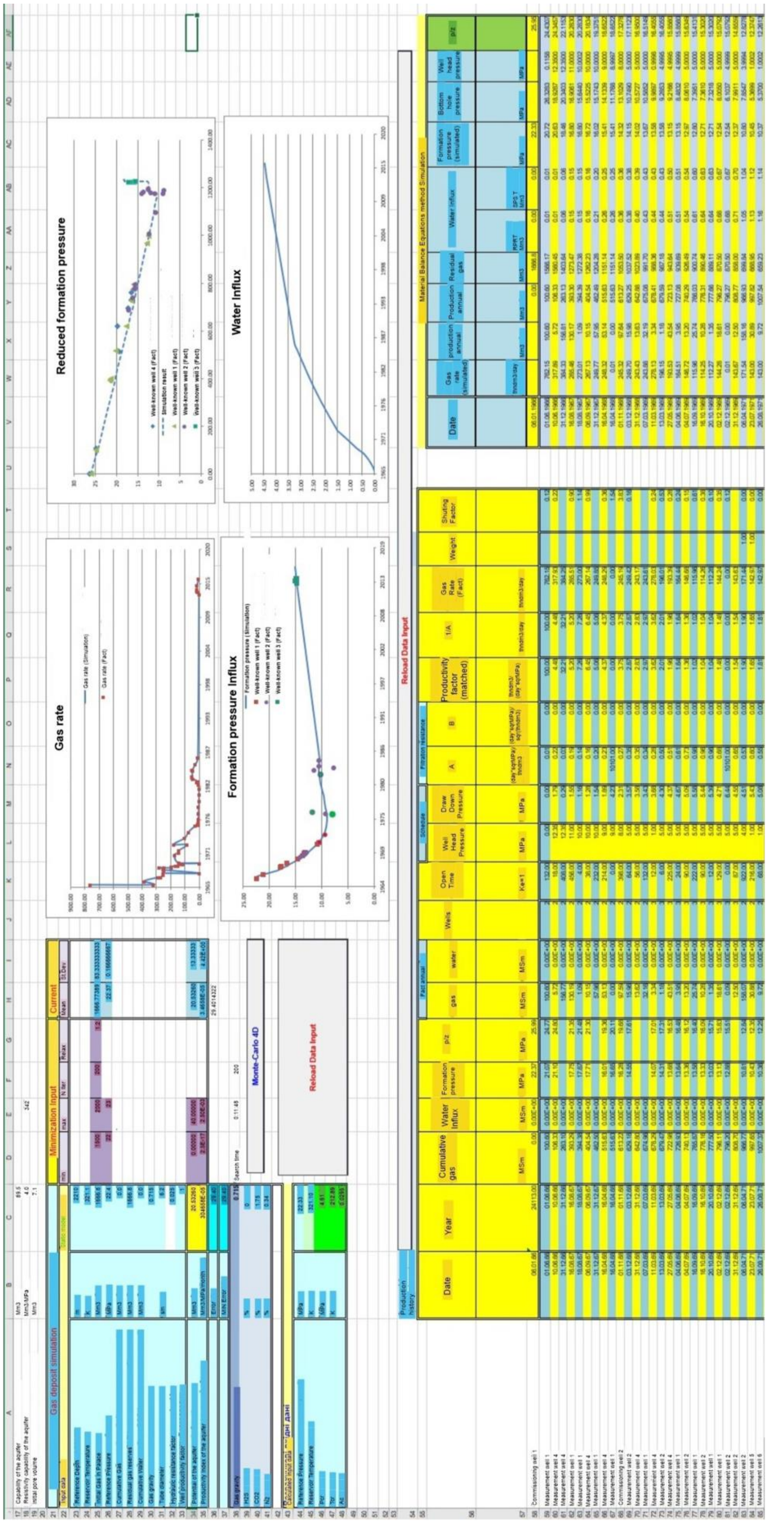

Figure 1 – Excel worksheet screenshot

Table 1 – The main characteristics of the model reservoir

Indicator	Dimension	Value
Outer border-radius	m	5000
Inner border-radius	m	1000
Gas reserves	mln m^3	2607.071
Piezo conductivity coefficient	m^2/day	2602.410
Aquifer potential	mln m^3	30.039
Fetkovich aquifer productivity index	mln m^3/(MPa·day)	$2.527 \cdot 10^{-4}$
Gas production rate from residual reserves	% per year	10

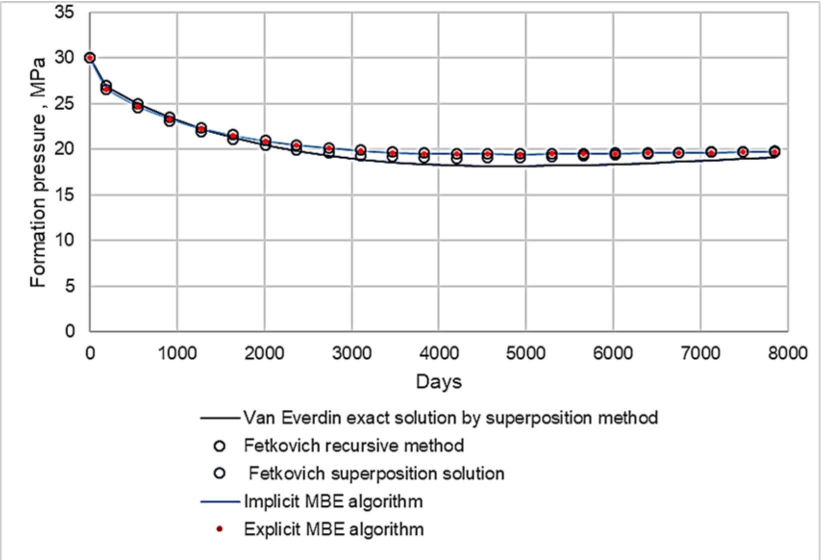

Figure 2 – Comparison of the calculated weighted average reservoir pressures for a circular reservoir R = 5

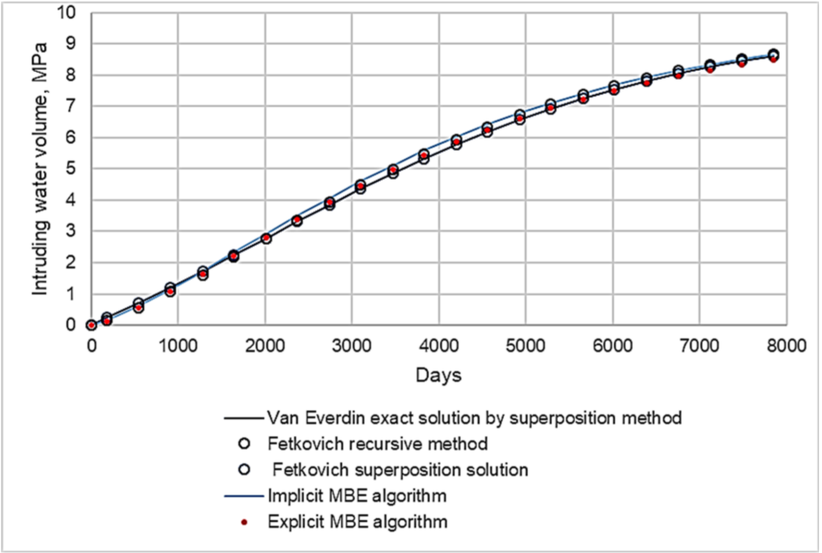

Figure 3 – Comparison of the calculated volumes of invading water for a circular formation R = 5

the radius of the gas content contour; $\gamma = \dfrac{k}{\mu \beta^*}$ is piezo conductivity coefficient; α_n are roots of the equation

$$J_1(\alpha_n R_a) Y_0(\alpha_n) - Y_1(\alpha_n R_a) J_0(\alpha_n) = 0;$$

$I_0^2(\alpha_n), I_0^2(\alpha_n R_a), J_0^2(\alpha_n), J_0^2(\alpha_n R_a), Y_0^2(\alpha_n), Y_0^2(\alpha_n R_a)$ are Bessel functions.

When testing the algorithms, the calculation results were compared by several methods:

calculation in the Mathcad system according to the principle of superposition with a kernel according to Van Evendinger and W. Harst (9);

calculation in the Mathcad system according to the principle of superposition with the kernel according to Fetkovich (4);

recursive calculation in the Mathcad system using the Fetkovich formula (5);

calculation in VBA Excel using an algorithm with an explicit scheme at a time step;

calculation in VBA Excel using an algorithm with an implicit scheme at a time step.

Figures 2 and 3 show the comparison of the calculation results according to various schemes of reservoir pressure dynamics in a hypothetical gas reservoir and the volume of water embedded in it from the aquifer.

The discrepancy in the calculation results between all methods is small, which means that there are no systematic errors in the algorithms. Thus, the root-mean-square discrepancy in calculating the pressure when using the principle of superposition with cores (9) and (4) is 0.68 MPa, and according to the calculation of embedded water volume – 0.11 million m^3. The use of recursive algorithms in comparison with the method of superposition with a kernel according to Van Evendinger and W. Harst (9) is not much higher. For the implicit pressure algorithm, the root mean square error is 0.89 MPa, and according to the calculation of the water influx, it is 0.16 million m^3; it is not much higher for the explicit algorithm – 0.90 MPa and 0.16 million m^3, respectively.

The problem of identifying the parameters of the material balance model

Like any mathematical model, the material balance function must adequately reflect a physical object. With regard to the possibilities and tasks posed in calculating the material balance, an adequate model should not contradict the existing geological ideas about the object, unreasonably contradict the estimates made by other methods, and describe with sufficient accuracy the actual historical dynamics of development indicators.

The practical use of material analysis equations to study the development of gas fields requires knowledge of four main parameters of the mathematical model (1) – (6): the initial drained gas reserves, the initial reservoir pressure in the reservoir, the potential and productivity index of the aquifer. This problem is solved by identifying them according to the actual dynamics of reservoir pressure in the reservoir during gas withdrawal.

The criterion in the identification procedure can be the quality of the description of the historical dynamics of changes in reservoir pressure, or quantitatively – the sum of the squares of deviations between N actual measurements of reservoir pressure and the pressure predicted for these dates by solving the basic equation of the material balance of the gas reservoir. Formally, the identification problem is reduced to functional minimization:

$$\min\left\{\sum_{i=1}^{N}\left[p_i - p_{pl}\left(Rzv_{ini}, p_{ini}, W_e, j_w, t_i\right)\right]^2\right\} \Rightarrow \begin{bmatrix} Rzv_{ini} \\ p_{ini} \\ W_e \\ j_w \end{bmatrix}. \quad (10)$$

The choice of an efficient extremum search algorithm is mainly determined by the topography of the response function. Figures 4 and 5 show the reliefs of the total squared error of approximation of the reservoir pressure dynamics for the gas deposit of the known field by the material balance equation. The pronounced non-monotonic nature of surfaces with the presence of many local minima excludes the use of gradient methods and predetermines the use of stochastic methods, in particular, random search by the Monte Carlo method.

As applied to the material balance calculation algorithm implemented in VBA Excel, the functional minimizer algorithm (10) is reduced to a sequential random change in the input parameters. After each sample, there is analyzed the value of the function. In the case of its decrease, the value of its mathematical expectation changes, and the value of the variance for the next sample decreases by a given relaxation factor.

Finding the minimum is like shooting in four-dimensional space. The mathematical expectation of the variables sets the center of the four-dimensional ball where the shot is directed, and their dispersion sets the radius of dispersion of the shot. A shot is the calculation of the objective function for certain values of the variables. The first shot gives the first initial approximation of the objective function. For each subsequent shot, one of the variables is sequentially randomly changed and there is determined a new value of the objective function. If the new value of the objective function (shot accuracy) is better than its previous value, the expectation of the variable is redefined to the current value and the variance for this variable decreases. Otherwise, the current values of the objective function, mean, and variance of the variable remain unchanged.

When generating a random value, it is assumed that the number of gas reserves, the initial reservoir pressure, and the potential of the aquifer obey the normal distribution law, and the productivity index of the aquifer obeys the logarithmically normal law. For the first sample, the values of the gas reserves, the initial reservoir pressure, which ensure the rapid convergence of the algorithm, give the already available data on the reservoir; for the potential of the aquifer, it is close to the value of the pore volume, and the production index is equal to zero. The initial values of variances are set by the assumed range in which the identified parameters are located.

An example of identifying the parameters of the material balance model for the known horizon of the field

The known horizon of the field as a separate development target (Figure 6) has been operated by 4 wells since 1999. In 2012, due to the flooding of the last well, the development of the facility was discontinued. 255.2 million m^3 of gas was produced from the deposit. It is estimated 416 million m^3 by the volumetric method.

The results of adaptation of the material balance model to the actual dynamics of reservoir pressure are shown in Figure 7. According to the criterion of minimizing the root-mean-square discrepancy between the predicted and actual dynamics of the mean reservoir pressure by the data show that the object was developed in a mixed-mode. Initial drained reserves are estimated at 334.1 million m^3 with an initial pressure in the reservoir of 12.27 MPa. The root-mean-square deviation between the measured and predicted reservoir pressures was 0.21 MPa. The material balance identifies a vast

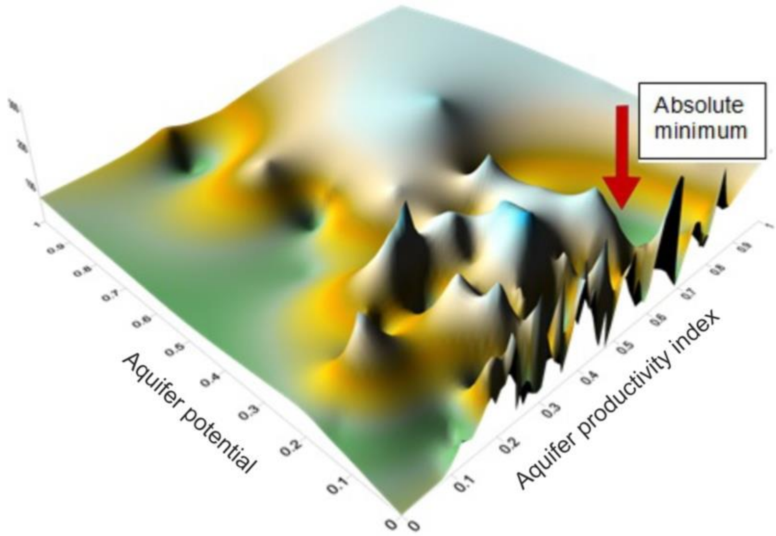

Figure 4 – The relief of the total square-law error of approximation of the reservoir pressure dynamics for the known gas reservoir of the field by the material balance equation depending on the potential and productivity index of the aquifer

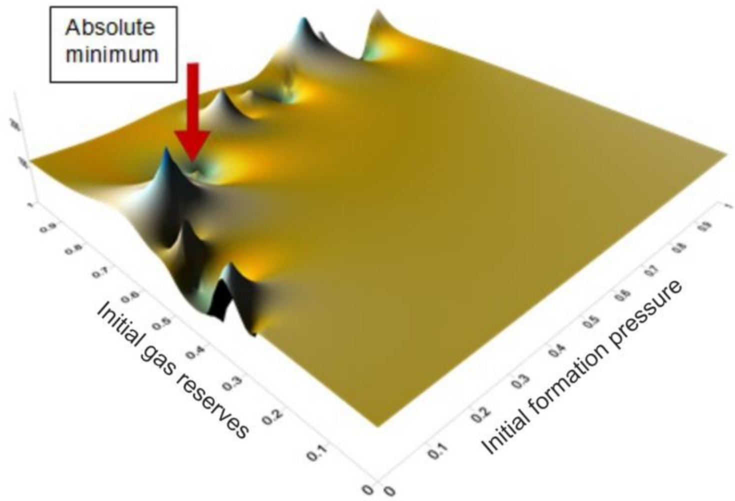

Figure 5 – **Relief of the total squared error of approximating the dynamics of reservoir pressure for the known gas reservoir by the material balance equation depending on the initial gas reserves and initial pressure**

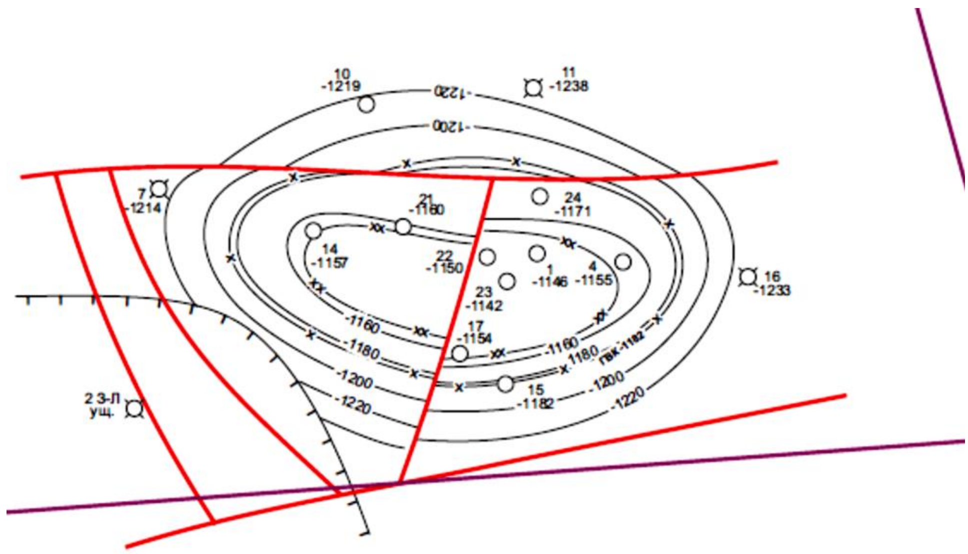

Figure 6 – Structural map along the roof of the known horizon

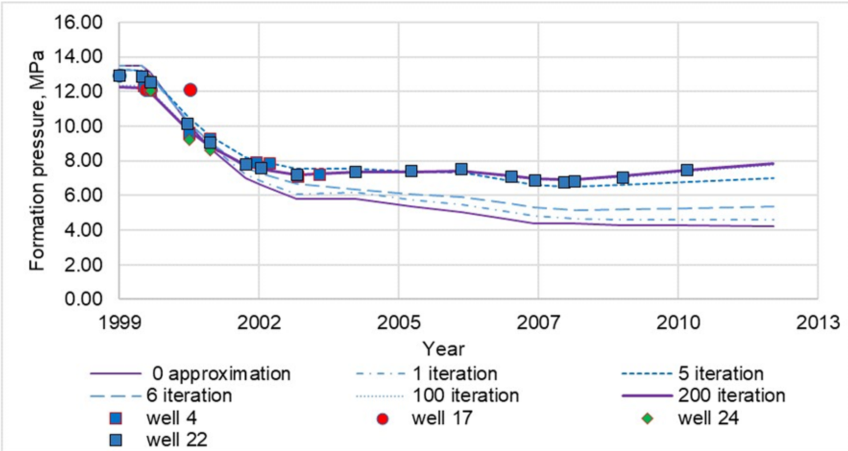

Figure 7 – Adaptation of MBE behind the formation pressure of the known horizon wells No. 4, 17, 22, 24

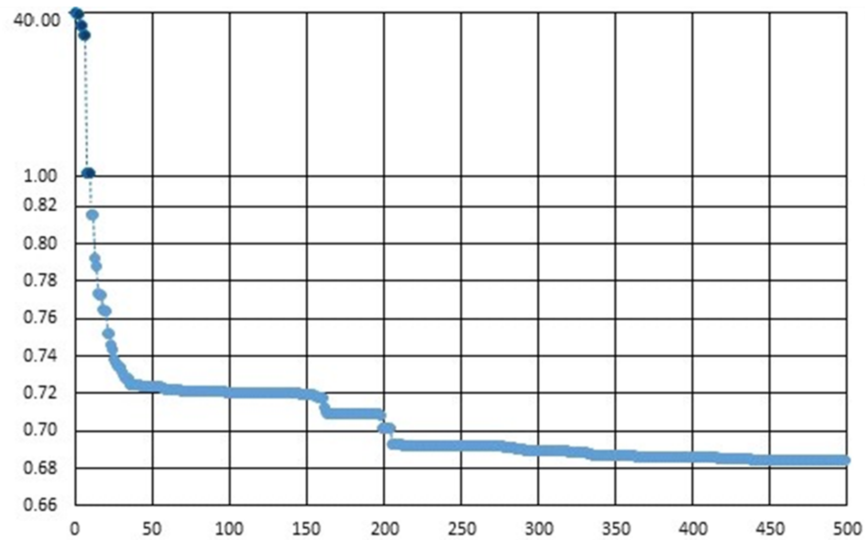

Figure 8 – Objective function dynamics depending on the number of minimizer samples

active contour area with a potential of 43.5 million m^3. The productivity index of the aquifer has a small value of $3.31 \cdot 10^{-5}$ million m^3/(MPa·day), which explains the late noticeable manifestation of its influence on the dynamics of reservoir pressure. Residual gas reserves are estimated at 78.98 million m^3 or 23.6 % of the original, which corresponds to the general idea of restrained gas volumes when gas is displaced by water [3].

Figure 8 illustrates the dynamics of the discrepancy between the calculated and measured values of reservoir pressure during the search for the best fit using the Monte Carlo method.

An example of identification of the parameters of the material balance model for the horizon of the known field

Development of the known deposit of the known field as a separate development target (Figure 9) has been operated by 3 wells 30, 101, and 112 since 1966. After the last one was flooded in 1986, the development of the facility was suspended. 1.225 billion m^3 of gas was produced from the deposit. According to the latter, before the termination of development, the reservoir pressure measurement in 1984 was 10.5 MPa.

More than 26 years later, with the introduction of a new well 207, the development of the known horizon facility was resumed. At the time of the well start-up, the pressure in the reservoir was partially restored and amounted to 14.7 MPa.

The dynamics of reservoir pressure in wells 30, 101, 112, and 207, despite the 26-year interruption in the operation of the facility, are in good agreement with each other and the model of the material balance of a gas reservoir in a mixed development mode.

According to the results of adaptation according to the criterion of the minimum total quadratic discrepancy between the actual and predicted dynamics of reservoir pressure, the drained reserves of the known horizons are estimated at 1.786 billion m^3 with an active and aquifer with a potential of 32.7 billion m^3 with a productivity index of $2.32 \cdot 10^{-5}$ million m^3/(MPa·day). During the reservoir development, 4.24 million m^3 (in reservoir conditions) of formation water invaded it. There is produced 75.1 % of gas, which is normal for a mixed-mode of development.

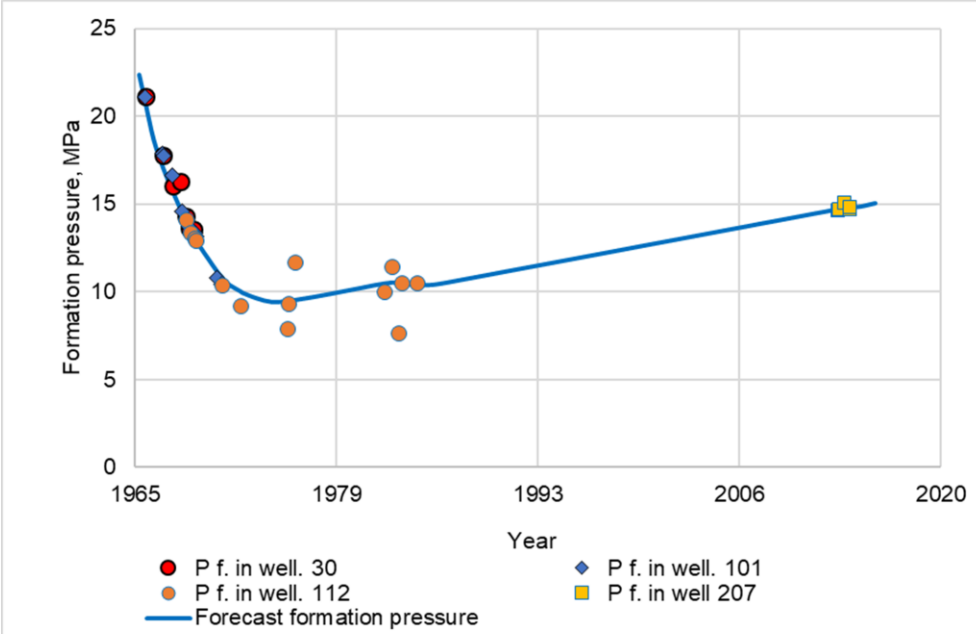

Figure 9 – Adaptation of the MBE behind the formation pressure of the known horizon wells 30, 101, 112

Conclusion

Thus, the algorithms for calculating the material balance and the algorithm for minimizing the discrepancies between the actual and calculated data, developed and implemented in VBA Excel, are a convenient tool for analyzing the development of gas fields, including those with a water-driven or mixed mode of development. The use of the identified parameters makes it possible to clarify the drained gas reserves, to assess the activity of formation waters and, as a result, to obtain an adequate reservoir model for predicting the main indicators of the gas development target.

References

[1] Fetkovich, MJ 1971, A simplified Approach to Water Influx Calculation-Finite Aquifer System, *JPT*, pp. 814–828.

[2] Van Everdingen, AF & Hurst, W 1949, The Application of the Laplace Transformation to Flow Problems in Reservoirs, *Trans. Aime*, pp. 305–324.

[3] Kondrat, RM 1992, *Gas Condensate Recovery*, Nedra, Moscow. [in Russian]

[4] Abed, HF, Abdullah, K & Naaem, W 2018, *Material balance and decline curve analysis to prediction the performance of reservoir using mbal softwere*, University of Missan, 80 p.

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Адаптація матеріального балансу газового покладу

Ю. Зарубін, М. Ґунда, П. Мамус

Науково-дослідний інститут нафтогазової промисловості НАК «Нафтогаз України;
м. Вишневе, Київська обл., Україна

Метою роботи є дослідження застосування методу матеріального балансу із використанням сучасних технологій для створення адекватної моделі газового пласта з активним поступленням води.

Проаналізовано основні рівняння матеріального балансу з урахування закону Менделєєва–Клапейрона для газів. Аналіз проведено з урахуванням активної водонапірної складової за рівняннями Фетковича. У роботі висвітлено проблематику оцінювання параметрів моделі матеріального балансу, оскільки останній не містить уявлення про геологічну структуру покладу. Розрахунки виконано на основі створених моделей покладу у MathCad та за допомогою VBA Excel.

Наведений у статті аналіз та приклади свідчать про доцільність застосування методу матеріального балансу з використанням VBA Excel для уточнення запасів газу, оцінки активності пластових вод та отримання адекватної моделі пласта.

Ключові слова: *газ, інженерні рішення, поклад, поступлення води, рівняння матеріального балансу, родовище.*