DOI: 10.31471/2311-1399-2021-2(16)-41-47

Modeling of distribution of pressure in a heterogeneous oil-bearing reservoir

*M. V. Lubkov, K. O. Mosiichuk**

National University «Yuri Kondratyuk Poltava Polytechnic»;
24, Pershotravneva Avenue, Poltava, 36011, Ukraine

Received: 04.07.2021 Accepted: 25.12.2021

Abstract

For the non-stationary problem of piezoconductivity, a numerical model of pressure distribution in an oil reservoir is proposed, which is based on a combination of well-known finite difference and finite element methods. The output data take into account the heterogeneity of the distribution of filtration characteristics across the reservoir. The pressure distributions obtained in this way correlate well with already-known ideas about reservoir operation at a late stage of development. A version of the calculation scheme with injection wells is considered. The results of calculations are visualized in the form of three-dimensional diagrams of reservoir pressure distribution. Recommendations are given for choosing a rational field development technology at a late stage of exploitation in order to increase the oil recovery rate in a heterogeneous reservoir.

Keywords: *numerical model, oil reservoir, pressure distribution.*

The problem of increasing the oil recovery rate remains relevant for fields at a late stage of development. To solve this problem, technologies for enhanced oil recovery are traditionally used [1–5], which are complemented by new materials and innovative combinations of previously known methods. The common feature of the mentioned technologies is that, ultimately, they are all aimed at changing the filtration parameters of the reservoir, such as permeability, porosity, viscosity, in order to obtain such combinations of them that would ensure an increase in the displacement of residual oil. However, for the effective use of these technologies, it is critical to have as full picture of oil filtration as possible in the supply circuit of the production well. The up-to-date question is: do all parameters equally affect the filtering intensity? Are there a number of factors that have priority in the operation of the reservoir in such conditions and on which, accordingly, the engineer's attention should be focused? According to the authors, each separate set of initial conditions must be considered separately and with the help of a comprehensive analysis by means of numerical modeling, parameters with the maximum level of influence on oil recovery should be identified. Based on the results of such an analysis, an effective reservoir stimulation strategy should be approved.

In general, numerical modeling methods allow solving various problems of an applied nature [6–8]:

enhanced oil recovery under various influences on the reservoir near the production well;

the general choice of the oil-bearing reservoir development system;

maintenance of optimal well production volumes;

determination of residual reserves and dead zones in the well supply circuit;

gradual analysis and decrease in the degree of developing risks and ensuring the strategy of operation of the active oil production wells system.

The oil and gas industry is the most active user of numerical modeling methods. Therefore, the reason is obvious: the impossibility of setting up a physical experiment directly over the reservoir. There is no such problem with the numerical reservoir model. We can judge the work of a real deposit by its behavior. The numerical model of the reservoir, in turn, is a computer program that uses numerical methods to solve the mathematical model of the reservoir (a system of nonlinear differential equations in derivative parts). Solving such a task necessarily requires the discretization of the computational domain. This process converts a system of nonlinear differential equations in derivative parts into a system of algebraic equations that give results at discrete points in space and moments of time. There are several methods of discretization implementation.

The most widely used in the oil and gas industry is the finite difference method. In it, a system of differential equations is written for a given point in space and moment of time. The selected moment of time (old, current, intermediate) leads to an explicit, implicit or Crank−Nicolson scheme [3]. As a result, the system of differential equations turns into a system of algebraic equations. After their linearization, a solution is obtained, which represents the distribution of pressure

* Corresponding author: karoleana2004@gmail.com

and saturation values in the reservoir, and the well production rate.

However, there remains a number of problems associated with the accuracy and adequacy of modeling of complex heterogeneous oil-bearing reservoirs in the conditions of real operation of oil-bearing fields. The finite-element-difference method proposed in this work combines the advantages of the finite-element method and the finite-difference method for solving the non-stationary problem of piezoconductivity, taking into account the heterogeneous distribution of filtration parameters inside the deformed reservoir and on its boundaries, and allows to adequately calculate the reservoir pressure distribution in real conditions of wells operation, which gives a number of advantages in comparison with existing methods.

The method assumes that the average thickness of the oil-bearing deformed porous formation is much smaller than the horizontal dimensions of the considered sector and uses a two-dimensional isotropic non-stationary piezoconductivity model [6, 9, 10]. In this case, the general formulation of the piezoconductivity problem, taking into account the condition of the intersection of the line sector in the Cartesian coordinate system (x, y), which is connected with the limits of the studied sector, has the form [10]:

$$\frac{\partial p}{\partial t} = \chi\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) + \gamma; \tag{1}$$

$$p(t=0) = p_0; \tag{2}$$

$$k\ grad\ p = \alpha(p - p_{cr}), \tag{3}$$

where (1) – equation of piezoconductivity; (2) – fixed pressure for each point; (3) – condition of fluid infiltration at the limits of the considered sector, when calculating oil filtration. The pressure p is given as a function of Cartesian coordinates and time; χ is the piezo-conductivity coefficient, which equals $k/(\eta m\beta_1 + \eta\beta_2)$; k is reservoir permeability; η is dynamic oil viscosity; m is porosity of the oil-bearing reservoir; β_1 is oil compression ratio; β_2 is the compressibility factor of the rock skeleton of an oil-bearing reservoir; γ is parameter of the intensity of production or injection of oil in the well; p_0 is initial reservoir pressure; α is oil infiltration coefficient at the limits of the considered sector; p_{cr} is pressure at the borders of the considered field.

In this work, the combined finite-element-defference method of M.V. Lubkov is used to solve the plane non-stationary problem of piezoconductivity, taking into account the heterogeneous distribution of filtration parameters inside the deformed conventional reservoir and at its limits [10]. The presented technique consists of the variational finite-element method and the finite-difference method.

First, the problem is formulated in a variational formulation. The functional of the variational formulation of the nonstationary plane problem of piezoconductivity in the general case has the form

$$I(p) = \frac{1}{2}\iint_S \left\{ k\left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2\right] + \right.$$
$$\left. +2\int_{p_0}^{p} \frac{k}{\chi}\frac{\partial p}{\partial t}dp - 2\gamma p \right\} dxdy - \tag{4}$$
$$-\frac{1}{2}\int_L \alpha\left(p - 2p_{cr}\right)pdl\ ,$$

where S is the area of the filtering side; L is the outline that covers area S; dl is the outline element.

To solve the non-stationary problem of piezoconductivity (1) – (3) a variational finite-element method is used, which provides a solution of the variational equation using the piezoconductivity functional (based on the Lagrange variational principle [8–10])

$$\delta I(P) = 0. \tag{5}$$

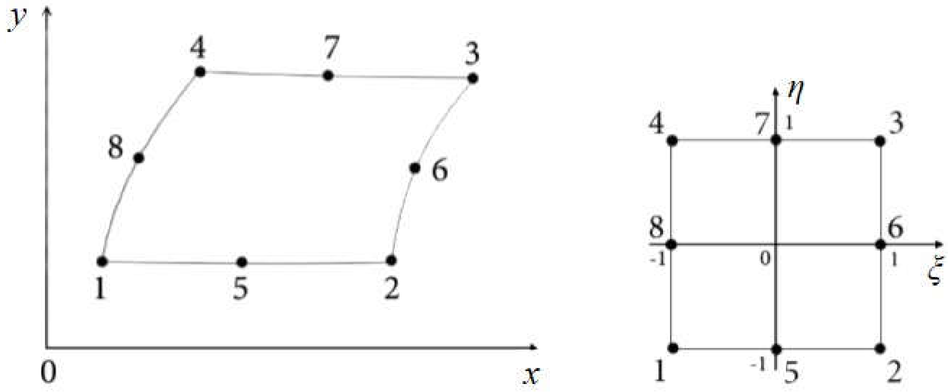
When solving the variational equation (5), an eight-node isoparametric quadrilateral finite element is used [10]. The Cartesian system is used as a global coordinate system, where all finite elements of the area S are combined *(x,y)*.

A normalized coordinate system is used as a local coordinate system, where approximation functions based on quadratic polynomials are determined within a finite element and numerical integration is carried out $(\xi,\ \eta)$ [10]. In this coordinate system, pressure, initial reservoir pressure, pressure at the borders of the field, oil infiltration coefficient at the borders of the field, as well as derivatives of the pressure along the coordinates are approximated in a known manner [10]:

$$x = \sum_{i=1}^{8} x_i\varphi_i;\ \ y = \sum_{i=1}^{8} y_i\varphi_i;$$
$$p = \sum_{i=1}^{8} p_i\varphi_i;\ \ p_0 = \sum_{i=1}^{8} p_{0i}\varphi_i;$$
$$p_{cr} = \sum_{i=1}^{8} p_{cri}\varphi_i;\ \ \alpha = \sum_{i=1}^{8} \alpha_i\varphi_i;$$
$$\frac{\partial p}{\partial x} = \sum_{i=1}^{8} p_i\Psi_i;\ \ \frac{\partial p}{\partial y} = \sum_{i=1}^{8} p_i\Phi_i;$$
$$\Psi_i = \frac{1}{|J|}\left(\frac{\partial\varphi_i}{\partial\eta}\frac{\partial y}{\partial\xi} - \frac{\partial\varphi_i}{\partial\xi}\frac{\partial y}{\partial\eta}\right); \tag{6}$$
$$\Phi_i = \frac{1}{|J|}\left(\frac{\partial\varphi_i}{\partial\xi}\frac{\partial x}{\partial\eta} - \frac{\partial\varphi_i}{\partial\eta}\frac{\partial x}{\partial\xi}\right),$$

where $J = \frac{\partial y}{\partial\xi}\frac{\partial x}{\partial\eta} - \frac{\partial y}{\partial\eta}\frac{\partial x}{\partial\xi}$ is the jacobian of the transition between systems *(x,y)* i $(\xi,\ \eta)$.

Based on the variational equation (5) and assuming that the nodal values of the pressure derivatives over time $\frac{dp_i}{dt}$ are known quantities and do not vary, let's formulate a system of differential equations for the n^{th} node of the p^{th} finite element in the form:



a) *b)*

a – Cartesian coordinate system (x, y); b – normalized coordinate system

Figure 1 – Eight-node isoparametric quadrilateral finite element for plane problems

$$\frac{\partial I_p}{\partial p_n}=\sum_{i=1}^{8}\{H_{ni}^{p}\frac{dp_i}{dt}+(A_{ni}^{p}+Q_{ni}^{p})p_i-Q_{ni}^{p}p_0^{i}\}- \\ -\gamma_n^p=0; \tag{7}$$

$$H_{ij}^{p}=\int_{-1}^{1}\int_{-1}^{1}\frac{k^p}{\chi^p}\varphi_i\varphi_j\left|J\right|d\xi d\eta;$$

$$A_{ij}^{p}=\int_{-1}^{1}\int_{-1}^{1}k^p(\Psi_i\Psi_j+\Phi_i\Phi_j)\left|J\right|d\xi d\eta;$$

$$Q_{ij}^{p}=\int_{L}\alpha\varphi_i\varphi_j dl;\gamma_i^p=\int_{-1}^{1}\int_{-1}^{1}\gamma^p\varphi_i\left|J\right|d\xi d\eta.$$

To solve the system of linear differential equations of the first order (7) under the initial conditions from (6) the finite difference method is used, in which the time derivative is approximated on the basis of an implicit difference scheme:

$$\frac{dp}{dt}=\frac{p(t+\Delta t)-p(t)}{\Delta t}. \tag{8}$$

By substituting expression (8) into system (7), we derive a system of linear algebraic equations:

$$\sum_{i=1}^{8}\{(\frac{1}{\Delta t}H_{ni}^{p}+A_{ni}^{p}+Q_{ni}^{p})p_i(t+\Delta t)- \\ -\frac{1}{\Delta t}H_{ni}^{p}p_i(t)-Q_{ni}^{p}p_0^{i}\}-\gamma_n^p=0, \tag{9}$$

$$n=1,2,...,8.$$

After integration over each finite element, a global system of linear algebraic equations is obtained, which makes it possible to determine the unknown pressure values at the moment of time $t+\Delta t$ through their values at the previous moment of time t. The solution of the global system of equations is carried out on the basis of the Gaussian elimination numerical method [10], as a result of which the pressure at all nodal points of the finite-element mesh of the studied sector at a given moment of time is determined.

Let's look at oil-bearing deposit with a production well measuring 90×90 m^2. Let's choose some typical average parameters of an oil-bearing reservoir [9, 10]: k =1D =10^{-12} m^2; m = 0.2; η = 10^{-3} Pa·s; β_1 = 10^{-9} Pa^{-1}; β_2 = 10^{-10} Pa^{-1}. In this case, the piezoconductivity coefficient χ=3.33 m^2/s. When modeling the distribution of pressures in the considered sector, let's assume that the initial pressure in the reservoir is 20 MPa, the average flow rate of the production well is 173 m^3 of oil per day. To minimize boundary effects during modeling, we will choose oil infiltration coefficients equal to 0.001 m.

The modeling results show that the process of setting the pressure within the considered section takes place a day after the start of its work. Fig. 1 *a, b* shows the distribution of pressure around the production well withing the filtration parameters specified above and the flow rate of the well 173 and 86.5 m^3 of oil per day, consequently.

Fig. 2 shows the distribution of pressure in the reservoir at different coefficients of oil permeability and viscosity in the close operation zone of the well (R_d < 5 m). Fig. 3 shows distribution of pressure in the neighbourhood of an operative well at different coefficients of oil permeability and viscosity in the remote operation zone of the well (R_d > 5 m). Fig. 4 shows distribution of pressure in the reservoir under the influence of an injection well of the same flow rate with different location options.

Analysis of the results of pressure distribution studies for the given numerical schemes (see Fig. 1–4) shows the approximately radial nature of oil filtration in the supply circuit of the production well. Here is an obvious impact of the filtration and capacity parameters of the reservoir on the flow rate of the well (see Fig. 2 and Fig. 3). For low permeability values of the reservoir, maintaining a sufficiently high level of oil recovery of the production well is connected with the use of injection wells (see Fig. 4). At the same time, it is advisable to use modern technologies that reduce waterlogging of the reservoir around the production well. It is obvious that the justification of the optimal conditions for oil recovery and, accordingly, oil production in each case is due to the selection of all parameters of the numerical model.

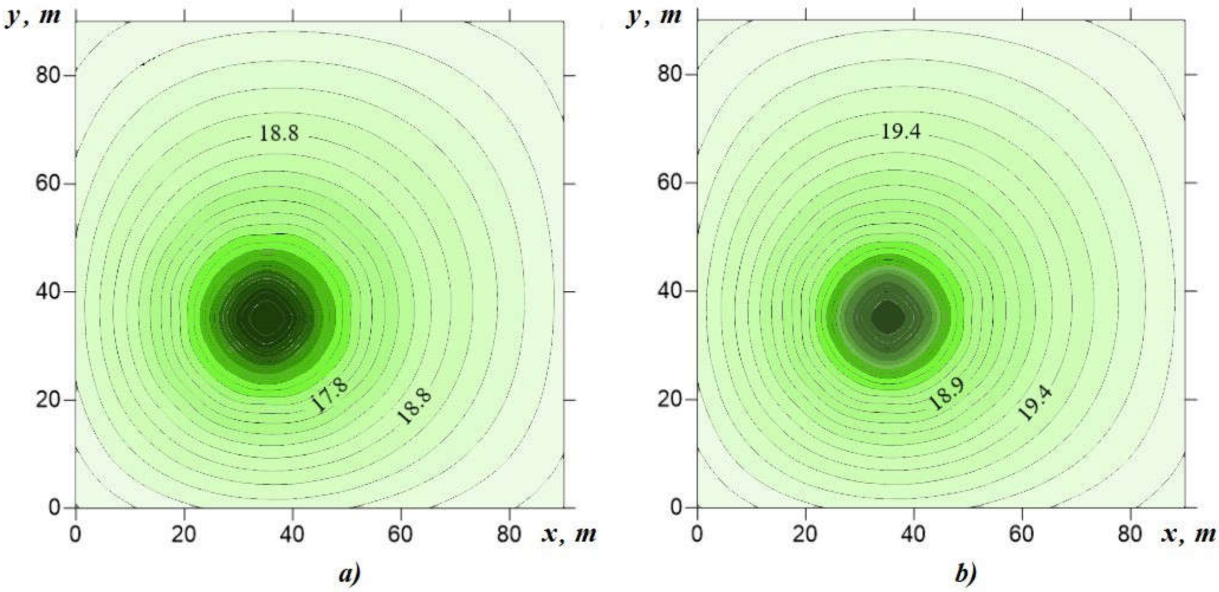


Figure 2 – Isolines of pressure (MPa) in the neighbourhood of the production well with flow rate of: *a*) 173 m^3/day; *b*) 86.5 m^3/day

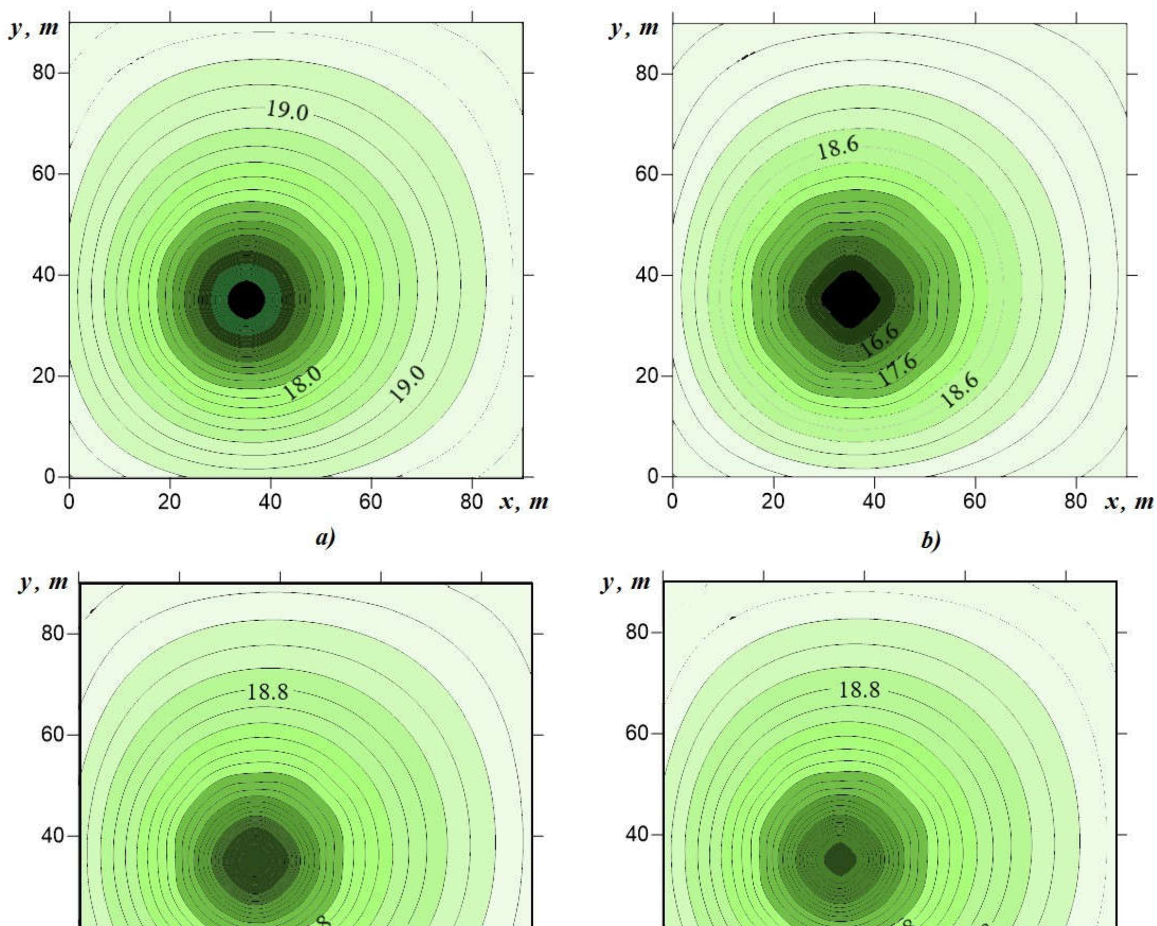


Figure 3 – Pressure isolines in the close zone (R_d < 5 m) of the production well at different coefficients of oil permeability and viscosity: *a*) k = 2 D; *b*) k = 0.5 D; *c*) $\eta = 2 \cdot 10^{-3}$ Pa·s; *d*) $\eta = 0.5 \cdot 10^{-3}$ Pa·s

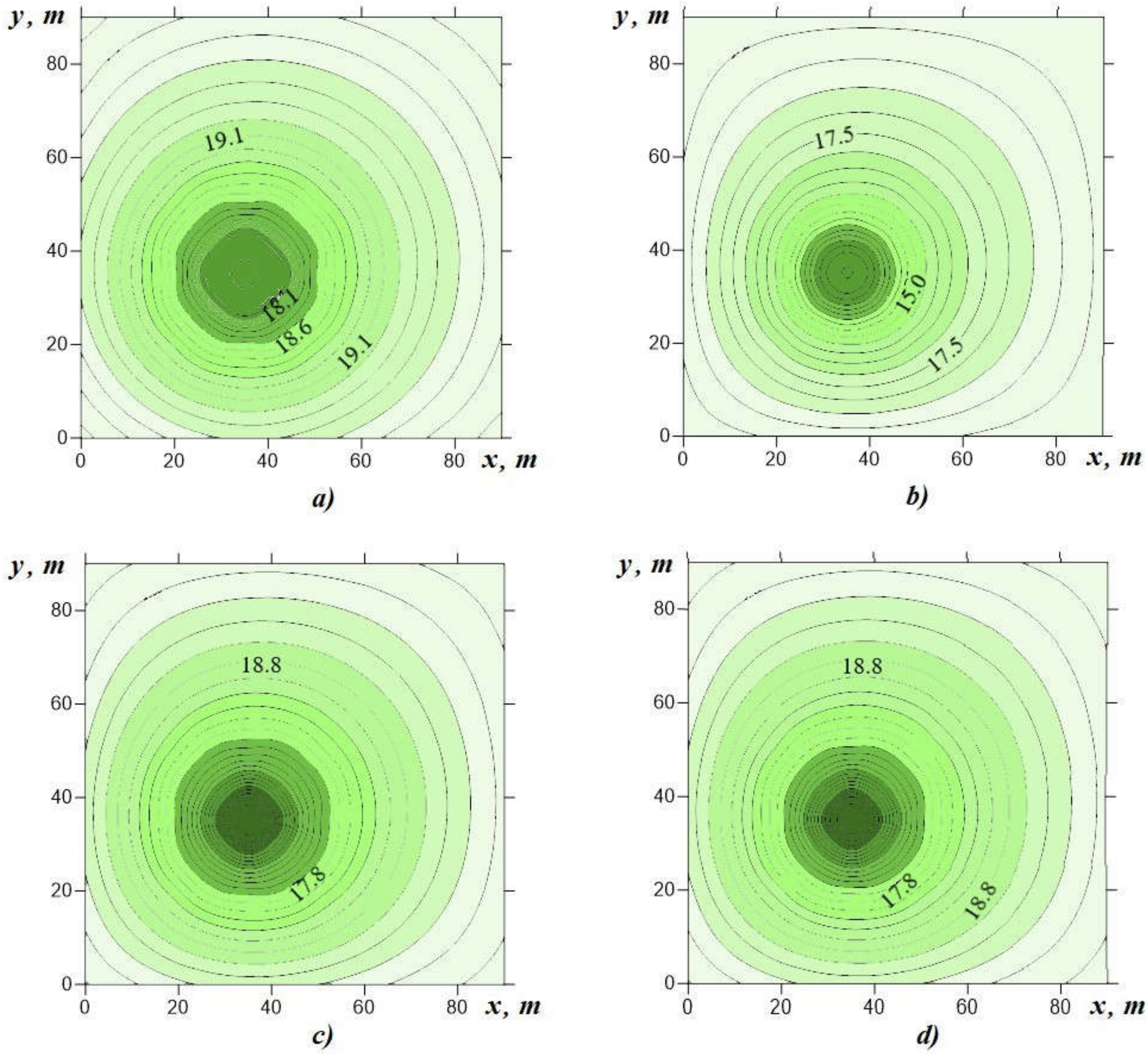


Figure 4 – Pressure isolines in the remote zone of the production well (R_d > 5 m) at different coefficients of oil permeability and viscosity: *a*) k = 2 D; *b*) k =0.5 D; *c*) $\eta = 2\cdot10^{-3}$ Pa·s; *d*) $\eta = 0.5\cdot10^{-3}$ Pa·s

Conclusions

A numerical model of oil filtration in a heterogeneous reservoir that is proposed in the research is based on the finite element and finite difference method. To check the adequacy of the obtained results, a non-stationary plane problem of piezoconductivity in compressible reservoirs was solved.

The modeling results show that the intensity of oil recovery mainly depends on the permeability of the reservoir, both in the bottomhole reservoir zone and at the boundary of the well supply circuit. Moreover, the influence of permeability on the circuit is greater compared to the bottomhole reservoir zone.

The analysis of the influence of the waterlogging fluid on the intensity of the filtration process around the production well confirms the feasibility of using injection wells for stable maintenance of a sufficiently high level of oil recovery at low permeability parameters of the reservoir.

References

[1] Abou-Kassem, JH, Farouq-Ali, SM & Islam, MR 2013, 'Petroleum Reservoir Simulations', *Elsevier*, vol. 1, iss. 2, pp. 45–67.

[2] Ohen, HA & Civan, F 1993, 'Simulation of formation damage in petroleum reservoirs', *SPE Advanced Technology Series*, vol. 1, iss. 1, pp. 27–35.

[3] Douglas, J, Furtado, F & Pereira, F 1997, 'On the numerical simulation of waterflooding of heterogeneous petroleum reservoirs', *Computational Geosciences*, vol. 1, iss. 2, pp. 155–190.

4. Douglas, J, Furtado, F & Pereira, F 1997, 'On the numerical simulation of waterflooding of heterogeneous petroleum reservoirs', *Computational Geosciences*, vol. 1, iss. 2, pp. 155–190.

[5] Wu, YS & Pruess, K 1988, 'A multiple-porosity method for simulation of naturally fractured petroleum reservoirs', *SPE Reservoir Engineering*, vol. 3, iss. 1, pp. 327–336.

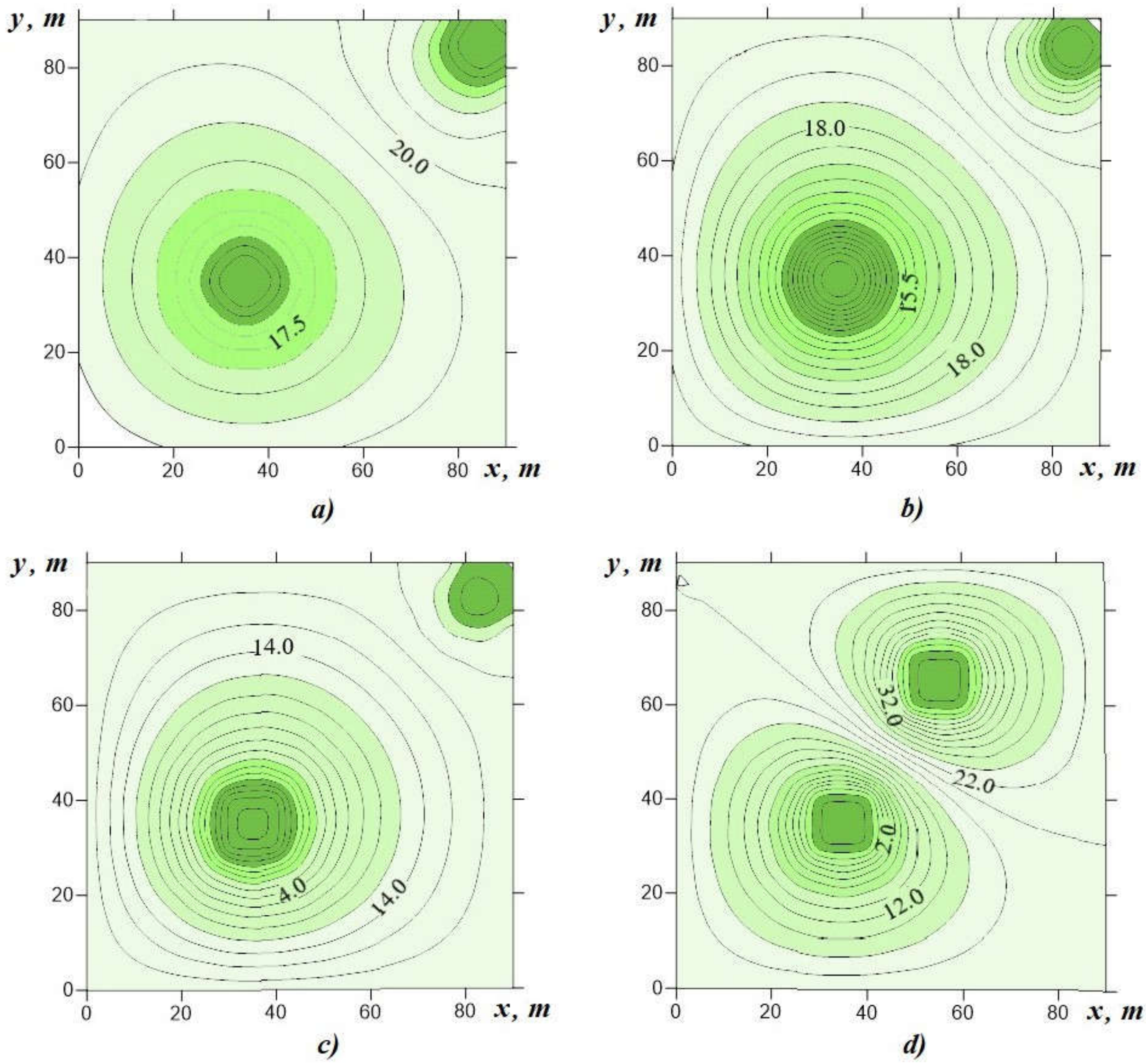


Figure 5 – Pressure isolines under the influence of an injection well of the same flow rate with different practical situations: *a*) with the filtering parameters specified above; *b*) k = 0.5 D (R_d > 5 m); *c*) k = 0.125 D (R_d > 5 m); *d*) k =0.125 D (R_d > 5 m)

[6] Aziz, Kh & Settari, Ye 2004, *Mathematical modeling of reservoir systems*, Institute of Computer Research, Moscow. [in Russian]

[7] Chen, Z, Huan, G & Ma, Y 2006, *Computational methods for multiphase flows in porous media*, Society for Industrial and Applied Mathematics, Philadelphia.

[8] Ertekin, T, Abou-Kassem, JH & King, GR 2001, *Basic applied reservoir simulation*. Texas: Richardson.

[9] Trangenstein, JA & Bell, JB 1989, 'Mathematical structure of the black-oil model for petroleum reservoir simulation', *SIAM Journal on Applied Mathematics*, vol. 49, iss. 3, pp. 749–783.

[10] Lybkov, MV, Zaharchuk, OO, Dmytrenko, VI & Petrash, OV 2021, 'Modeling of efficient pressure in heterogeneous oil-bearing reservoirs', *Bulletin of NTU "KhPI". Series: Chemistry, chemical technologies and ecology",* no. 2 (6), pp. 23–29. doi: 10.20998/2079-0821.2021.02.10. [in Ukrainian]

УДК 553.982

Моделювання розподілу тиску в неоднорідному нафтоносному пласті

М. В. Лубков, К. О. Мосійчук

Національний університет «Полтавська політехніка імені Юрія Кондратюка»;
просп. Першотравневий, 24, Полтава, 36011, Україна

Для нестаціонарної задачі п'єзопровідності запропоновано чисельну модель розподілу тиску у нафтоносному пласті, яка основана на поєднанні відомих методів кінцевих різниць та кінцевих елементів. Вихідні дані враховують неоднорідність розподілу фільтраційних характеристик по пласту. Отримані таким способом розподіли тиску добре корелюються із відомими уявленнями про роботу пласта на пізній стадії розробки. Розглянуто варіант розрахункової схеми з нагнітальними свердловинами. Результати розрахунків візуалізовано у формі тривимірних діаграм розподілу пластового тиску. Наведено рекомендації щодо вибору раціональної технології розробки родовища на пізній стадії експлуатації з метою підвищення коефіцієнту вилучення нафти в неоднорідному пласті.

Ключові слова: *нафтоносний пласт, розподіл тиску, чисельна модель.*