To the issue of strength assessment of the pipeline-composite bandage system

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Abstract

In case of the long-term operation of pipelines under the influence of alternating loads and degradation of service characteristics of material there originate residual stresses and strains that can lead to the loss of its bearing capacity in pipes. One of the ways to increase the structural strength of pipelines is to strengthen the pipe by banding, which results in the need to develop methods for calculating the structural parameters of the pipeline – bandage system. This paper presents a mathematical model to estimate the strength of the pipeline-composite bandage system, based on the improved theory of thin shells with regard to shear and compression pliability of material. The solution allows us to simulate the stress state of a finite-length pipeline based on the given original characteristics of the pipes` material, composite, operational loads and effects.

Key words: *bearing capacity, composite bandage, residual strains, stress state.*

We apply the equation of the refined Tymoshenko shells theory to study the stress state of the pipe with residual strains caused by some factors taking into account shear and compression pliability of material [1]. The circular cross section pipe with the wall thickness $2h$ is simulated with a finite cylindrical shell of the radius R, and its median surface is attributed to the lines of curvature α_1 and α_2 (in this case they are generating and directing lines of the shell). In the future we will use dimensionless coordinates α, φ ($\alpha = \alpha_1$ is a dimensionless length of the generatrix, $\varphi = \alpha_2$ is the central angle of the arc of the directing line). The axis z is directed towards the outer normal to the mid-surface. The origin of coordinates is set at a point in the middle of the annular area $-b \le \alpha \le b$ of the tube (Fig. 1) in the localized area of residual strains.

Due to a priori knowledge of its qualitive behavior there can be used a semi-empirical model. In this case the distribution of residual strains along axis α of the cylindrical shell can be assumed as symmetrical, and therefore

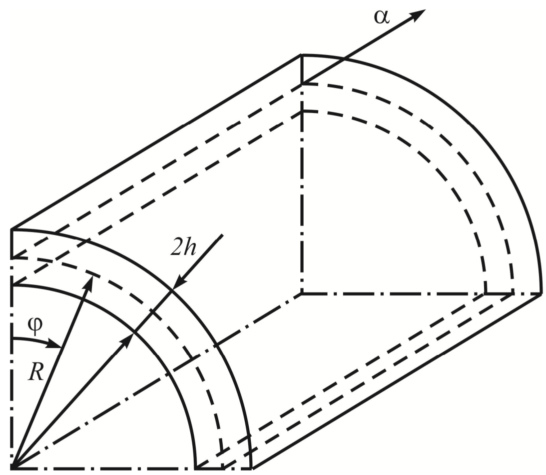

Figure 1 – Analytical model of the pipe element

$$\varepsilon_{12}^0 = \varepsilon_{23}^0 = \varepsilon_{13}^0 = 0,\ \varepsilon_{11}^0 = k\varepsilon_{22}^0,\ \varepsilon_{22}^0 = \varepsilon_{23}^{*}\psi(\alpha),$$
$$\varepsilon_{33}^0 = -(\varepsilon_{11}^0 + \varepsilon_{22}^0) = -(1+k)\varepsilon_{22}^0,\ k_{ii}^0 = 0. \quad (1)$$

Indices 1, 2, 3 correspond to the coordinate axes α, φ, z. The values ε_{ij}^0, k_{ii}^0 of the formula (1) mean the average components of the distortion tensor [1]. They are calculated as follows:

$$\varepsilon_{ij}^0 = \frac{1}{2h}\int_{-h}^{h} e_{ij}^0 dz, \quad i, j = 1,2,3,$$

$$k_{ii}^0 = \frac{3}{2h^3}\int_{-h}^{h} e_{ii}^0 z dz, \quad i = 1,2.$$

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The function $\psi(\alpha)$, which describes the area of residual strains, is set in the following way

$$\psi(\alpha) = 1 - a(\alpha/b)^2 - (1-a)(\alpha/b)^4, 0 \le \alpha \le b, \quad (2)$$

where b is a relative half-width of the area of residual strains. The function is $\psi(\alpha) = 0$ in the area $b < \alpha \le L$. We apply the following symbols: $\alpha = x/R$ is a dimensionless axial coordinate, ε_0^* is the maximum residual strains, a, k are some stable parameters, $2l$ is the shell length, $L = l/R$.

When deriving the basic equations we give components of a complete strain $e_{ij}\,(i, j = 1,2,3)$ in any point of the shell in the following way [2]:

$$e_{ij} = e_{ij}^e + e_{ij}^0, \quad (3)$$

where e_{ij}^0 are components of a residual strain tensor, e_{ij}^e are components of additional strain tensor that ensure the continuity of components of a complete strain tensor e_{ij} and they are related to components of the strain tensor σ_{ij} by the relationships of Hooke's law.

In case of orthotropic material and according to the accepted model these relationships are the following [1]

$$e_{11}^e = \frac{1}{E_1}\sigma_{11} - \frac{\nu_{12}}{E_2}\sigma_{22} - \frac{\nu_{13}}{E_2}\sigma_{33}, \; e_{13}^e = \frac{1}{G_{13}}\sigma_{13},$$

$$e_{22}^e = -\frac{\nu_{21}}{E_1}\sigma_{11} + \frac{1}{E_2}\sigma_{22} - \frac{\nu_{23}}{E_3}\sigma_{33}, \; e_{23}^e = \frac{1}{G_{23}}\sigma_{23}, \quad (4)$$

$$e_{33}^e = -\frac{\nu_{31}}{E_1}\sigma_{11} - \frac{\nu_{32}}{E_2}\sigma_{22} + \frac{1}{E_3}\sigma_{33}, \; e_{12}^e = \frac{1}{G_{12}}\sigma_{12}.$$

Due to the symmetry the following dependencies are performed for the coefficients of equations (4):

$$E_2\nu_{21} = E_3\nu_{31}, \; E_3\nu_{31} = E_1\nu_{13}, \; E_3\nu_{32} = E_2\nu_{23}.$$

Let us write the dependence (2) in another form:

$$\sigma_{11} = E_{10}(e_{11}^e + \nu_{12}e_{22}^e) + \lambda_1, \; \sigma_{33} = G_{13}e_{13}^e,$$

$$\sigma_{22} = E_{20}(e_{22}^e + \nu_{21}e_{11}^e) + \lambda_2, \; \sigma_{33} = G_{23}e_{13}^e, \quad (5)$$

$$\sigma_{33} = E_{30}(e_{33}^e + \lambda_1 e_{11}^e + \lambda_2 e_{22}^e), \; \sigma_{12} = G_{12}e_{12}^e.$$

Therefore as a result of substitution of (1) into (3) we obtain

$$\sigma_{11} = E_{10}(e_{11} + \nu_{12}e_{22}) + \lambda_1\sigma_{33} - E_{10}(e_{11}^0 + \nu_{12}e_{22}^0),$$

$$\sigma_{22} = E_{20}(e_{22} + \nu_{21}e_{11}) + \lambda_2\sigma_{33} - E_{20}(e_{22}^0 + \nu_{21}e_{11}^0),$$

$$\sigma_{33} = E_{30}(e_{33} + \lambda_1 e_{11} + \lambda_2 e_{22}) - E_{30}(e_{33}^0 + \lambda_1 e_{11}^0 + \lambda_2 e_{22}^0), \quad (6)$$

$$\sigma_{13} = G_{13}e_{13} - G_{13}e_{13}^0,$$

$$\sigma_{23} = G_{23}e_{23} - G_{23}e_{23}^0,$$

$$\sigma_{12} = G_{12}e_{12} - G_{12}e_{12}^0.$$

The following symbols are applied here:

$$E_{10} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \; E_{20} = \frac{E_2}{1-\nu_{12}\nu_{21}},$$

$$E_{30} = \frac{E_3(1-\nu_{12}\nu_{21})}{1-\nu_{21}\nu_{12} - \nu_{31}\nu_{13} - \nu_{23}\nu_{32} - 2\nu_{31}\nu_{12}\nu_{13}}, \quad (7)$$

$$\lambda_1 = \frac{\nu_{31}\nu_{12} + \nu_{32}}{1-\nu_{12}\nu_{21}}\frac{E_1}{E_2}, \; \lambda_2 = \frac{\nu_{21}\nu_{13} + \nu_{23}}{1-\nu_{12}\nu_{21}}\frac{E_2}{E_3}.$$

In case of transversely isotropic material the expressions (7) for the elastic constants get simplified and take the following form:

$$E_{10} = E_{20} = E_0 = \frac{E}{1-\nu^2}, \; \nu_{13} = \nu',$$

$$E_3 = E', \; E_{30} = E_0' = \frac{(1-\nu)E'}{1-\nu-2(\nu')^2 E/E'},$$

$$\lambda_1 = \lambda_2 = \lambda = \frac{E}{E'}\frac{\nu'}{1-\nu}, \quad (8)$$

$$G_{13} = G_{23} = G', \; G_{12} = G = \frac{E}{2(1+\nu)}.$$

For the isotropic material based on (8) we obtain:

$$E' = E, \; \nu' = \nu, \; G' = G = \frac{E}{2(1+\nu)},$$

$$E_0 = \frac{E}{1-\nu^2}, \; \lambda = \frac{\nu}{1-\nu}, \; E_0' = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}. \quad (9)$$

If the chosen approach [1] is $\{0,1\}$ we obtain correlations for determining the stress state of a pipe caused by residual strains. Balanced equations have the following form:

$$\frac{\partial}{\partial\alpha}N_1 + \frac{\partial}{\partial\varphi}N_{12} + 2R\sigma_1^- = 0,$$

$$\frac{\partial}{\partial\varphi}N_2 + \frac{\partial}{\partial\alpha}N_{12} + Q_2 + 2R\sigma_1^- = 0,$$

$$\frac{\partial}{\partial\alpha}M_1 + \frac{\partial}{\partial\varphi}M_{12} - RQ_1 + 2R\sigma_1^+ = 0, \quad (10)$$

$$\frac{\partial}{\partial\varphi}M_2 + \frac{\partial}{\partial\alpha}M_{12} - RQ_2 + 2hR\sigma_2^+ = 0,$$

$$\frac{\partial}{\partial\alpha}Q_1 + \frac{\partial}{\partial\varphi}Q_2 - N_2 + 2R\sigma_3^- = 0.$$

In this case, the ratio for the generalized efforts N_1, N_2, N_{12}, Q_1, Q_2, and moments M_1, M_2, M_{12} (characteristics of the state of stress of the middle surface) are the following:

$$N_1 = \frac{B_1}{R}\left(\frac{\partial u}{\partial\alpha} + \nu_{12}\frac{\partial v}{\partial\varphi}\right) + 2h\lambda_1\sigma_3^+ - B_1\left(\varepsilon_{11}^0 + \nu_{12}\varepsilon_{22}^0\right),$$

$$N_2 = \frac{B_2}{R}\left(\frac{\partial v}{\partial\varphi} + \nu_{12}\frac{\partial u}{\partial\alpha}\right) + 2h\lambda_2\sigma_3^- - B_{21}\left(\varepsilon_{22}^0 + \nu_{21}\varepsilon_{11}^0\right),$$

$$M_1 = \frac{D_1 h}{R}\left(\frac{\partial\gamma_1}{\partial\alpha} + \nu_{12}\frac{\partial\gamma_2}{\partial\varphi}\right) + \frac{2}{3}\lambda_1 h^2\sigma_3^+ - D_1\left(k_{11}^0 + \nu_{21}k_{11}^0\right),$$

$$M_2 = \frac{D_2 h}{R}\left(\frac{\partial\gamma_2}{\partial\alpha} + \nu_{21}\frac{\partial\gamma_1}{\partial\varphi}\right) + \frac{2}{3}\lambda_2 h^2\sigma_3^- - D_2\left(k_{22}^0 + \nu_{21}k_{11}^0\right),$$

$$N_{12} = \frac{D_{12}}{R}\left(\frac{\partial u}{\partial \varphi} + \frac{\partial v}{\partial \alpha}\right) - D_{12}\varepsilon_{12}^{0},$$

$$M_{12} = \frac{D_{12}h}{R}\left(\frac{\partial \gamma_1}{\partial \varphi} + \frac{\partial \gamma_2}{\partial \alpha}\right) - D_{12}k_{12}^{0},$$

$$Q_1 = \Lambda_1'\left(\gamma_1 + \frac{1}{R}\frac{\partial w}{\partial \alpha}\right) - \frac{2}{3}h\sigma_1^{+} - \Lambda_1'\varepsilon_{13}^{0}, \quad (11)$$

$$Q_2 = \Lambda_2'\left(\gamma_2 + \frac{1}{R}\frac{\partial w}{\partial \varphi}\right) - \frac{2}{3}h\sigma_2^{+} - \Lambda_2'\varepsilon_{23}^{0},$$

where

$$B_1 = \frac{2E_1 h}{1-\nu_{12}\nu_{21}}, \quad B_2 = \frac{2E_2 h}{1-\nu_{12}\nu_{21}},$$

$$D_1 = \frac{1}{3}B_1 h, \quad D_2 = \frac{1}{3}B_2 h, \quad (12)$$

$$B_{12} = 2hG_{12}, \quad D_{12} = \frac{1}{3}B_{12}h,$$

$$\Lambda_1' = \frac{2}{3}hk'G_{13}, \quad \Lambda_2' = \frac{2}{3}hk'G_{23}, \quad k' = \frac{5}{6}.$$

The upper index "+" or "–" correspond to the external and internal surfaces of a pipe.

The ratio is obtained in the same way as in the monograph [2] for the shells with finite-sliding stiffness, the stress state of which is caused by the given distortion tensor. If $\sigma_i^- = 0$ or $\sigma_i^+ = 0$ (i = 1,2,3) we obtain a key system of equations for cylindrical shells, the stress state of which is caused by the disturbed (defective) stress-strain state on the basis of (10) and (11). In axisymmetric case (the values are independent of the variable $\varphi, v = 0, \gamma_2 = 0$) the system takes the following form for a given distribution of residual strains (12):

$$\frac{d^2\bar{u}}{d\alpha^2} + \nu\left(\frac{d^2 w}{d\alpha}\right) = \frac{d}{d\alpha}\left(\varepsilon_{11}^{0} + \nu\varepsilon_{22}^{0}\right),$$

$$\frac{d^2\gamma}{d\alpha^2} - \frac{R^2\Lambda'}{Dh}\left(\gamma + \frac{d\bar{w}}{d\alpha}\right) = 0, \quad (13)$$

$$\frac{d^2\bar{w}}{d\alpha^2} + \frac{d\gamma}{d\alpha} - \frac{B}{\Lambda'}\left(\bar{w} + \nu\frac{d\bar{u}}{d\alpha}\right) = \frac{B}{\Lambda'}\left(\varepsilon_{22}^{0} + \nu\varepsilon_{11}^{0}\right),$$

where $\bar{u} = u/\left(R\varepsilon_0^*\right)$, $\bar{w} = w/\left(R\varepsilon_0^*\right)$ are dimensionless values of axial and normal displacement of a shell, $\gamma = \gamma_1/\varepsilon_0^*$ is a turn angle of the normal element to its mid-surface.

The values B, D, Λ' from (13) for the isotropic material are determined by the following formulas:

$$B = \frac{2Eh}{1-\nu^2}, \; D = \frac{2Eh^2}{3\left(1-\nu^2\right)} = \frac{1}{3}Bh, \quad (14)$$

$$\Lambda' = \frac{2}{3}Gh = \frac{Eh}{3(1-\nu)},$$

where E, ν are the elastic modulus and Poisson's ratio.

It should be noted that the system (13) is equivalent to the corresponding system of a power problem, if we assume that a dummy load vector $\Phi(\alpha)$ is attached to the shell. In a certain case it has the following form:

$$\Phi(\alpha) = \left(\frac{d}{d\alpha}\left(\varepsilon_{11}^{0} + \nu\varepsilon_{22}^{0}\right), 0, \frac{B}{\Lambda'}\left(\varepsilon_{22}^{0} + \varepsilon_{11}^{0}\right)\right). \quad (15)$$

The system of differential equations (13) is calculated under the the following boundary conditions:

$$\bar{u} = 0, \; \gamma = 0, \; \frac{d\bar{w}}{d\alpha} = 0 \; (\alpha = 0),$$

$$\bar{w} = 0, \; \frac{d\gamma}{d\alpha} = 0, \; \gamma + \frac{d\bar{w}}{d\alpha} = 0 \; (\alpha = L).$$

If $\alpha = 0$ symmetry boundary conditions are met, and if $\alpha = L$ free edge boundary conditions are met.

Comprehensive consideration of residual strains during calculation of the stress-strain state of tubes (cylindrical shells) by analytical methods is an extremely difficult task [2–4]. To assess the impact of residual strains upon the stress state of a pipe we apply a developed numerical and analytical method for solving linear boundary problems in case there are systems of ordinary differential equations with constant coefficients [5]. Under the conditions of a given distribution of residual strains the formulated problem can be solved as the boundary problem (13), (15). According to the scheme of the method we present the system of differential equations (13) in Cauchy normal form:

$$\frac{dz}{d\alpha} = Az(\alpha) + \Phi(\alpha),$$

where

$$z = \left(z_1, z_2, ..., z_6\right)^{\mathrm{T}} = \left(\bar{u}, \gamma, \bar{w}, \frac{d\bar{u}}{d\alpha}, \frac{d\gamma}{d\alpha}, \frac{d\bar{w}}{d\alpha}\right)^{\mathrm{T}},$$

$$\Phi(\alpha) = \left(0,0,0,0, f_1(\alpha), f_2(\alpha)\right),$$

$$f_1(\alpha) = -(k+\nu)\varepsilon_0^*\left(\frac{2a}{b^2}\alpha + \frac{4(1-\alpha)}{b^4}\alpha^3\right), \quad (16)$$

$$f_2(\alpha) = \frac{B}{\Lambda'}(1+\nu k)\varepsilon_0^*\varphi(\alpha) \quad \text{if } 0 < \alpha < \mathrm{b},$$

$$f_1(\alpha) = f_2(\alpha) = 0 \quad \text{if } \alpha > b.$$

Matrix $A = \left[a_{ij}\right], \; i, j = \overline{1,5}$ of the system (16) has only nonzero elements defined by geometric and elastic characteristics of the shell:

$$a_{14} = a_{25} = a_{36} = 1, a_{46} = -\nu,$$

$$a_{52} = a_{56} = \frac{R^2\Lambda'}{Dh}, a_{63} = \frac{B}{\Lambda'}, a_{64} = \frac{B}{\Lambda'}\nu, a_{65} = -1. \quad (17)$$

Based on the symbols from (17) we can show the conditions (14) in the following way:

$$z_1 = 0, \; z_2 = 0, \; z_6 = 0 \; (\alpha = 0),$$

$$z_3 = 0, \; z_5 = 0, \; z_2 + z_6 = 0 \; (\alpha = L). \quad (18)$$

Having applied a numerical and analytical method [6], we obtain an approximate solution of a boundary problem (16), (18) in an analytical form:

$$z_i(\alpha) = \sum_{k=1}^{6} C_k g_{ik}(\alpha) + \int_0^{\alpha}\left[g_{i4}(\alpha-\tau) f_1(\tau) + g_{i6}(\alpha-\tau) f_2(\tau)\right]d\tau,$$

$$i = 1, 2, ..., 6. \quad (19)$$

The functions of $g_{ij}(\alpha)$ are the elements of a matrizer that are defined by the elements of the matrix *A* of the system (16). We apply the classic method of matrizer approximation by the matrix row according to the following formulas

$$g_{ii}(\alpha)=1+\sum_{k=1}^{M}a_{ij}^{(k)}\frac{\alpha^k}{k!},$$
$$g_{ij}(\alpha)=\sum_{k=1}^{M}a_{ij}^{(k)}\frac{\alpha^k}{k!}\quad (i\neq j, i, j=1,2,...,6), \qquad (20)$$

where $a_{ij}^{(k)}$ are the elements of a matrix A^k, M is the number of elements in a row of a matrix that approximates the matrizer while numerical calculation.

Unknown constants in a formula (19) are defined from the system of linear equations, obtained as a result of meeting the conditions of (18). Having fulfilled these conditions at $\alpha = 0$ and taking into account (20) we calculate $C_1 = 0$, $C_2 = 0$, $C_6 = 0$. Under the conditions of (18) at $a = L$, taking into account that the functions $f_1(\alpha)$, $f_2(\alpha)$ are set according to the formulas (17), we obtain a system of algebraic equations for calculating the constants C_3, C_4, C_5 :

$$C_3\bar{g}_{33}+C_4\bar{g}_{34}+C_5\bar{g}_{35}=I_1,$$
$$C_3\bar{g}_{53}+C_4\bar{g}_{54}+C_5\bar{g}_{55}=I_2, \qquad (21)$$
$$C_3(\bar{g}_{23}+\bar{g}_{63})+C_4(\bar{g}_{24}+\bar{g}_{64})+C_5(\bar{g}_{25}+\bar{g}_{65})=I_3,$$

where

$$\bar{g}_{ij}=g_{ij}(L),\ I_1=-\int_0^b(g_{34}(L-\tau)f_1(\tau)+g_{36}(L-\tau)f_2(\tau))d\tau,$$

$$I_2=-\int_0^b(g_{54}(L-\tau)f_1(\tau)+g_{56}(L-\tau)f_2(\tau))d\tau,$$

$$I_3=-\int_0^b(g_{24}(L-\tau)f_1(\tau)+g_{26}(L-\tau)f_2(\tau))d\tau-$$
$$-\int_0^b(g_{64}(L-\tau)f_1(\tau)+g_{66}(L-\tau)f_2(\tau))d\tau.$$

Numerical realization of the algorithm for computing integrals is performed by the Gauss method [6]. Having applied the solution of (19) and the conditions of (18) at $\alpha = 0$ the following formula is obtained for determining flexture of a tube:

$$\bar{\omega}(\alpha)=z_3(\alpha)=C_3g_{33}(\alpha)+C_4g_{34}(\alpha)+C_5g_{35}(\alpha)+$$
$$+\int_0^a(g_{35}(\alpha-\tau)f_1(\tau)+g_{36}(\alpha-\tau)f_2(\tau))d\tau, \qquad (22)$$

where $0\le\alpha\le b$ is the half-width of the zone of residual strains localization. The lengths of deflection shell at the area of $b<\alpha\le L$ are determined by the formula (22), but it is necessary to replace the upper limit of integration by b in the integral. Similarly, we determine other unknown quantities of the system of equations (16). It should be noted that the scheme for finding the solution by the offered method is the same for any distribution of residual strains in the pipe.

According to the chosen theoretical model and the way of the disturbed (defective) stress-strain state we make the calculation of stresses in any point of the pipe by the following formulas:

$$\bar{\sigma}_{11}(\alpha,\bar{z})=\frac{d\bar{u}}{d\alpha}+\nu\bar{w}+\frac{h}{R}\frac{d\gamma}{d\alpha}\bar{z}+\lambda\bar{\sigma}_{33}(\alpha,\bar{z})-(k+\nu)\psi(\alpha),$$
$$\bar{\sigma}_{22}(\alpha,\bar{z})=\bar{w}+\nu\left(\frac{d\bar{u}}{d\alpha}+\frac{h}{R}\frac{d\gamma}{d\alpha}\bar{z}\right)-$$
$$-(1+\nu k)\psi(\alpha)+\lambda\bar{\sigma}_{33}(\alpha,\bar{z}), \qquad (23)$$
$$\bar{\sigma}_{33}(\alpha,\bar{z})=\frac{3\bar{E}}{E_0}(1-\bar{z}^2)\left(\lambda\left(\frac{d\bar{u}}{d\alpha}+\bar{w}\right)-(1-\lambda)(1+k)\psi(\alpha)\right),$$
$$\bar{\sigma}_{13}(\sigma,\bar{z})=\frac{5}{6}(1-\nu)\left(\gamma+\frac{d\bar{w}}{d\alpha}\right)(1-\bar{z}^2),$$

where $\bar{\sigma}_{ij}=\frac{\sigma_{ij}}{E_0\varepsilon_0^*}$, $\lambda=\frac{\nu}{1-\nu}$, $E_0=\frac{E}{1-\nu^2}$, $\bar{E}=\frac{(1-\nu)E}{(1+\nu)(1-2\nu)}$ – for the isotropic material, $\bar{z}=z/h$, $-h\le z\le h$.

It should be noted that if $\lambda=0$ (compression is not included) we obtain the results that correspond to Tymoshenko model [6]. The solutions of the problem for determining the stress state of transversely isotropic shells for some cases of axially symmetric and sustainable distribution of residual strains are presented in [3]. Let us provide and comment on the results for this case of residual strains distributions (Fig. 1):

$$e_{22}^0(x,z)=\varepsilon^0\left(1+\frac{\mu_0}{h}z\right)S(x+x_0), \qquad (24)$$

where $S(t)=\begin{cases}1, t>0\\1, t<0\end{cases}$ is Heaviside function, μ_0 is the parameter which characterizes the change in the value of e according to the membrane thickness; $2x_0$ is the width of the disturbed (defective) stress-strain state. For example, distribution (24) corresponds to the longitudinal shrinkage of the annular zone (due to structural changes, operating conditions, environmental influences, technological processes, etc.).

A received solution to the problem helps to understand the stress state of a finite-length pipe based on a given finite initial disturbed state (value and residual strains distribution). This allows one to calculate the strength based on given operating loadings and assess resource of a pipe, reinforced with composite bandage, compared to the conventional (standard) pipe without defects, damages and structural changes.

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До оцінки міцності системи трубопровід – композитний бандаж

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Під час довготривалої експлуатації трубопроводів внаслідок впливу знакозмінних навантажень та деградації службових характеристик матеріалу у трубах виникають залишкові напруження і деформації, що можуть призводити до втрати її несучої здатності. Одним із напрямків підвищення конструктивної міцності трубопроводів є зміцнення тіла труби бандажуванням, що зумовлює необхідність розвитку методів розрахунку конструктивних параметрів системи трубопровід – бандаж. У статті наведена математична модель оцінки міцності системи трубопровід – композитний бандаж, яка ґрунтується на уточненій теорії тонких оболонок із урахуванням податливості матеріалу бандажа на зсув та обтиснення. Отриманий розв’язок дає можливість моделювати напружений стан трубопроводу скінченної довжини для заданих початкових характеристик матеріалу труб і композиту, а також експлуатаційних навантажень і впливів.

Ключові слова: *залишкові деформації, композитний бандаж, напружений стан, несуча здатність.*