# Calculating the steady-state operations of profile gas pipelines with high and ultrahigh pressures 

M.V. Lurie *<br>Gubkin Russian State University of Oil and Gas;<br>65, prospect Leninskii, Moscow, 119991, Russian Federation

Received: 28.06.2013 Accepted: 11.10.2013


#### Abstract

The pipelines, the working conditions of which are significantly different from the conditions of the majority of the existing systems are considered in this article. In this article, the gas pipelines with large difference in elevation that operate at high ( $5-15 \mathrm{MPa}$ ) and ultrahigh pressures (up to $25-35 \mathrm{MPa}$ ) are taken into the account. First of all, deep and gas pipelines that overcome the high mountain passes are considered. The theory and algorithm for calculating the steady-state operations of such pipelines are presented.


Key Words: differential equations, equation of state, gas pipeline "South Stream", high and ultra-high-pressures, JouleThomson coefficient, numerical algorithm, profile of the gas pipeline.

Up to now, various methods for calculating gas pipelines of high pressure are known and tested [1-4]. However, many of those methods contain simplifying assumptions, which in most cases, is justified, and presents quite significant results. Moreover, in recent years, gas pipelines, working conditions, which are very different from many existing pipelines. In this case, we deal with gas pipelines with large difference in elevation fans that operate at high ( $5-15 \mathrm{MPa}$ ) and ultrahigh pressures (up to $25-35 \mathrm{MPa}$ ) in a wide temperature range and complexity of the heat transfer. These deepwater gas pipelines are of the following pipeline types: "Blue Stream", "South Stream", "The North European Gas Pipeline" etc., as well as gas pipelines that overcome high mountain passes. In extreme conditions under which these pipelines are being operated, the factors, which in ordinary cases are small or negligible can not be neglected. In this paper, the theory and algorithm of termo-hydraulic calculating of steady mode pipelines, and in particular, the use of numerical methods are methodologically and consistently presented.

## 1. Basic Equations for Steady Pipeline Gas Flow

To calculate the steady flow of compressible gas pipeline in the area, the following equations are used:

[^0]the continuity equation:
$$
\frac{d}{d x}(\rho v S)=0
$$
from which we receive the following equation:
\[

$$
\begin{equation*}
Q=\rho v S=\text { idem. } \tag{1}
\end{equation*}
$$

\]

that implies constant mass flow $Q$ along the pipeline. Since the density $\rho$ of the gas decreases as the pressure drop, then from the equation (1) we can conclude that in the case of constant cross-sectional area $\mathrm{S}=$ idem the gas velocity $v$ of the gas from the start to the end increases;
equation of motion:

$$
\begin{equation*}
\rho v \frac{d v}{d x}=-\frac{d p}{d x}-\frac{4}{d} \tau_{w}-\rho g \frac{d z}{d x}, \tag{2}
\end{equation*}
$$

whereas $\tau_{w}=(\lambda / 8) \rho v^{2}$ is the shear stress on the inner friction surface of the conduit; $d$ is the diameter of pipeline; $g$ is the gravity acceleration; $z(x)$ is the pipeline profile; $d z / d x=\sin \alpha(x)$ is the sine of the angle to the horizontal axis of the conduit.
the total energy balance equation:

$$
\begin{equation*}
Q \frac{d}{d x}\left(\frac{\alpha_{k} v^{2}}{2}+J\right)=\pi d q_{n}-Q g \frac{d z}{d x} \tag{3}
\end{equation*}
$$

whereas $q_{n}$ is the heat of the flow per unit of surface of the conduit. If the dependence of the enthalpy of the pressure and the temperature T is used in this equation, putting $J=J(p, T)$, and also if to assume that the gas heat exchange with the environment follows Newton's heat transfer $q_{n}=K_{T}\left(T-T_{a m b}\right)$, where $K_{T}$ is the heat transfer coefficient and $T_{\text {amb }}$ is the ambient temperature, we will receive the following exation:

Table 1 - Values of the ratio $\left(C_{p}-C_{V}\right) / R$

| Pressure, | Temperature $T, \mathrm{~K}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p, \mathrm{MPa}$ | 270 | 280 | 290 | 300 | 310 | 320 | 330 | 340 |
| 1.00 | 1.14 | 1.13 | 1.11 | 1.10 | 1.09 | 1.09 | 1.08 | 1.07 |
| 3.00 | 1.48 | 1.42 | 1.38 | 1.34 | 1.30 | 1.27 | 1.26 | 1.22 |
| 5.00 | 1.94 | 1.81 | 1.70 | 1.61 | 1.55 | 1.49 | 1.44 | 1.40 |
| 9.00 | 3.22 | 2.80 | 2.50 | 2.28 | 2.11 | 1.97 | 1.86 | 1.77 |
| 12.00 | 4.04 | 3.50 | 3.08 | 2.78 | 2.53 | 2.34 | 2.17 | 2.05 |
| 16.00 | 4.28 | 3.89 | 3.54 | 3.21 | 2.95 | 2.70 | 2.53 | 2.36 |
| 20.00 | 3.99 | 3.79 | 3.57 | 3.33 | 3.11 | 2.91 | 2.72 | 2.57 |
| 24.00 | 3.64 | 3.56 | 3.41 | 3.26 | 3.10 | 2.95 | 2.79 | 2.66 |
| 30.00 | 3.22 | 3.19 | 3.13 | 3.05 | 2.96 | 2.85 | 2.76 | 2.66 |

Table 2 - The Values of the coefficient $\mathrm{D}_{*}(\mathrm{p}, \mathrm{T})$ the Joule-Thomson, K/MPa

| Pressure, | Temperature $T, \mathrm{~K}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p, \mathrm{MPa}$ | 270 | 280 | 290 | 300 | 310 | 320 | 330 | 340 |  |
| 1.00 | 5.39 | 5.13 | 4.60 | 4.53 | 4.01 | 4.05 | 3.60 | 3.32 |  |
| 3.00 | 5.36 | 4.98 | 4.64 | 4.32 | 4.04 | 3.75 | 3.62 | 3.29 |  |
| 5.00 | 5.18 | 4.82 | 4.47 | 4.15 | 3.91 | 3.64 | 3.42 | 3.23 |  |
| 9.00 | 4.39 | 4.13 | 3.89 | 3.66 | 3.43 | 3.23 | 3.05 | 2.87 |  |
| 12.00 | 3.45 | 3.37 | 3.25 | 3.11 | 2.97 | 2.82 | 2.67 | 2.55 |  |
| 16.00 | 2.25 | 2.33 | 2.36 | 2.34 | 2.29 | 2.22 | 2.16 | 2.07 |  |
| 20.00 | 1.40 | 1.53 | 1.61 | 1.66 | 1.68 | 1.68 | 1.66 | 1.63 |  |
| 24.00 | 0.86 | 0.98 | 1.08 | 1.14 | 1.20 | 1.22 | 1.23 | 1.23 |  |
| 30.00 | 0.39 | 0.48 | 0.56 | 0.62 | 0.68 | 0.72 | 0.75 | 0.77 |  |

$$
\begin{gathered}
\frac{d}{d x}\left(\frac{\alpha_{k} v^{2}}{2}\right)+\left(\frac{\partial J}{\partial T}\right)_{p} \frac{d T}{d x}+\left(\frac{\partial J}{\partial p}\right)_{T} \frac{d p}{d x}= \\
=-\frac{\pi d K_{T}}{Q}\left(T-T_{a m b}\right)-g \frac{d z}{d x} \\
\text { Denoting } \quad(\partial J / \partial T)_{p}=C_{p} ; \quad(\partial J / \partial p)_{T}=-C_{p} D_{*}
\end{gathered}
$$ and accepting the Coriolis coefficient $\alpha_{k}$ which equals 1 , we represent the equation (3) in the following way:

$$
\begin{align*}
& v \frac{d v}{d x}+C_{p}\left(\frac{d T}{d x}-D_{*} \frac{d p}{d x}\right)= \\
= & -\frac{\pi d K_{T}}{Q}\left(T-T_{a m b}\right)-g \frac{d z}{d x} \tag{4}
\end{align*}
$$

In this case, the heat capacity at constant pressure, $D_{*}(p, T)-$ a Joule-Thomson coefficient:
the equation of state of a real gas:

$$
\begin{equation*}
p=Z(p, T) \rho R T, \tag{5}
\end{equation*}
$$

where $Z$ is the compressibility factor; $R$ is the gas constant.

With the help of the equation of the state of a real gas, the main thermodynamic coefficients can be expressed in terms of the function $Z(p, T)$. In particular, the expression for that connection of heat capacity at the constant pressure and volume for a real gas acquires the following look [5]:

$$
C_{p}-C_{V}=-\frac{T}{\rho^{2}}\left(\frac{\partial p}{\partial T}\right)_{\rho}\left(\frac{\partial \rho}{\partial T}\right)_{p}
$$

Expressing the derivatives in this expression, through pressure and temperature with the help of the equation of state $p=Z(p, T) \rho R T$, we will obtain:

$$
\begin{equation*}
C_{p}(p, T)=C_{V}+R \frac{\left[Z+T\left(\frac{\partial Z}{\partial T}\right)_{p}\right]^{2}}{Z-p\left(\frac{\partial Z}{\partial p}\right)_{T}} \tag{6}
\end{equation*}
$$

where $C_{V}$ is the heat capacity of gas at the constant volume. For the perfect gas $Z \equiv 1$, there-fore $C_{p}-C_{V}=R$. For a real difference in gas $C_{p}-C_{V}>R$. Ratio values $Z(p, T)$ are calculated by formula (6) using the function $Z(p, T)$ for the natural gas; they are presented in Table 1.

The Joule-Thomson coefficient $D_{*}(p, T)$ is represented by the formula, where heat $C_{p}(p, T)$ is given by the equeation (6) [3]. Using the equation of $p=Z(p, T) \rho R T$ as the state of gas, we obtain the following expression:

$$
\begin{equation*}
D_{*}(p, T)=\frac{1}{C_{p}(p, T)} \frac{T}{\rho Z}\left(\frac{\partial Z}{\partial T}\right) \tag{7}
\end{equation*}
$$

For the perfect gas $Z \equiv 1$, thus $D_{*} \equiv 0$.
The values of the coefficient $D_{*}(p, T)$ are calculated by the formula (7) using $Z(p, T)$ of natural gas and are presented in Table 2.

It should be noted that the Joule-Thomson effect can change its direction of action to reversed when the derivative $(\partial Z / \partial T)_{p}$ changes the sign from positive to negative. Based upon the known charts $Z(p, T)$ of natural gas, we deal with the pressure of $35-40 \mathrm{MPa}$.

Equations (1)-(7) are the basis for calculation of the steady non-isothermal gas in the pipeline with an arbitrary profile.

## 2. Method of Calculating the Established Gas Pipelines Modes

If in the equation (2) of the gas movement to express the shear stress $\tau_{w}$ on the inner surface of the pipe according to the formula $\tau_{w}=(\lambda / 8) \rho v^{2}$, the system of equations for the calculation of the steadystate regimes of gas pipelines can be represented as:

$$
\left\{\begin{array}{l}
\rho v \frac{d v}{d x}+\frac{d p}{d x}=-\lambda \frac{1}{d} \frac{\rho v^{2}}{2}-\rho g \frac{d z}{d x} \\
\frac{d}{d x}\left(\frac{v^{2}}{2}+J\right)=-\frac{\pi d K_{T}}{Q}\left(T-T_{a m b}\right)-g \frac{d z}{d x}  \tag{8}\\
p=Z(p, T) \rho R T .
\end{array}\right.
$$

Since the mass flow rate is $Q=i d e m$, the the velocity $v$ is not an independent variable and is determined by $v=Q /(\rho S)$. If from the second equation (8) to subtract the first one, we received a 2-system of ordinary differential equations

$$
\left\{\begin{array}{l}
a_{1}(p, T) \frac{d p}{d x}+b_{1}(p, T) \frac{d T}{d x}=c_{1}(p, T, x),  \tag{9}\\
a_{2}(p, T) \frac{d p}{d x}+b_{2}(p, T) \frac{d T}{d x}=c_{2}(p, T)
\end{array}\right.
$$

for 2 unknowns - pressure $p(x)$ and temperature $T(x)$. Here,

$$
\begin{gathered}
a_{1}(p, T)=\frac{1}{(Q / S)^{2}}-\frac{R T}{p^{2}}\left[Z-p\left(\frac{\partial Z}{\partial p}\right)_{T}\right], \\
b_{1}(p, T)=\frac{\partial}{\partial T}\left(\frac{1}{\rho}\right)_{p}=\frac{R}{p}\left[Z+T\left(\frac{\partial Z}{\partial T}\right)_{p}\right], \\
c_{1}(p, T, x)=-\frac{1}{(Q / S)^{2}}\left[\frac{\lambda}{2 d} \frac{(Q / S)^{2}}{\rho}+\rho g \frac{d z}{d x}\right], \\
a_{2}(p, T)=\left(\frac{\partial J}{\partial p}\right)_{T}-\frac{1}{\rho}=-\frac{R T}{p}\left[Z+T\left(\frac{\partial Z}{\partial T}\right)_{p}\right], \\
b_{2}(p, T)=\left(\frac{\partial J}{\partial T}\right)_{p}=C_{p}(p, T)= \\
=C_{V}+R\left[Z+T\left(\frac{\partial Z}{\partial T}\right)_{p}\right] /\left[Z-p\left(\frac{\partial Z}{\partial p}\right)_{T}\right], \\
c_{2}(p, T)=-\frac{\pi d K_{T}}{Q}\left(T-T_{a m b}\right)+\frac{\lambda}{2 d} \frac{(Q / S)^{2}}{\rho^{2}} .
\end{gathered}
$$

Whereas the coefficient $Z(p, T)$ of compressibility is considered to be a known function of its arguments.

If the main determinant $\Delta$ of the system of equations (9) is different from zero, i.e., $\Delta=a_{1} b_{2}-a_{2} b_{1} \neq 0$, it can be uniquely solved for the derivatives $d p / d x$ and $d T / d x$ using the well-known Cramer's rule:

$$
\left\{\begin{array}{l}
\frac{d p}{d x}=\frac{\Delta_{1}}{\Delta}  \tag{10}\\
\frac{d T}{d x}=\frac{\Delta_{2}}{\Delta}
\end{array}\right.
$$

where $\Delta_{1}=c_{1} b_{2}-c_{2} b$ and $\Delta_{2}=a_{1} c_{2}-a_{2} c_{1}$. Here, the right sides of equations (8) are unknown functions of $p, T$ and $x$, and the mass flow of gas $Q$ is included into them as a constant parameter.

The system of equations (10) can be integrated into any of the standard methods, for example, by a numerical method of Runge-Kutta method or by a simpler Euler lines method. Both of these methods are included in practically almost any mathematical application package of computer programs.

The greatest practical interest are the solutions of the following two tasks.

Task 1. Find the distribution of pressure $p(x)$ and temperature $T(x)$ along the participating pipeline if the initial section of a predetermined pressure $p_{0}=p(0)$ and temperature $T_{0}=T(0)$, and the mass flow of the gas $Q$ is unknown. Find also the gas pressure $p_{L}$ at the end of pipeline part.

Task 2. Find the mass flow of the gas $Q$ if the initial and final part of the pipeline section have the pressure set as $p_{0}=p(0)$ and $p_{L}=p(L)$, and the set temperature of the gas is $T_{0}=T(0)$ at the beginning of the plot. Find the gas temperature $T_{L}$ at the gas-end of the pipeline.

The first task solution, which is in accordance with the mathematical terminology of the initial Cauchy problem is constructed by numerical methods, which are mentioned above. The second task is not the primary, because its conditions are set on the edges of the field of integration of $x \in[0, L]$, i.e. and in sections $x=0$ and $x=L$ such tasks are called boundary.

## 3. Iterative Algorithm of Numerical Calculations

The solution of the second task (the boundary one) can be reduced to the solution of the first (initial Cauchy problem) if to waive the condition $p(L)=p_{L}$ in time, however, instead of that, it is significant to consider, in this case, known $Q$ as a mass flow rate. Then, in the initial section $x=0$ of the conduit portion, the pressure $p_{0}$ and temperature $T_{0}$ of the gas will be known, and the mass flow $Q$ will not be as an unknown quantity anymore. Consequently, it is possible to use the method
of integration that was used to solve the task 1. Of course, within such an approach the pressure $p(L)$ at the end of pipeline section, generally speaking, will not be equal to the predetermined value $p_{L}$, so it is necessary to vary the flow rate $Q$ to achieve the desired equality.

The essence of the iterative algorithm, called zeroing algorithm consists in the following. First, the interval of $(0, Q)$ values is designated, which can have massive gas consumption: $0 \leq Q \leq Q_{\max }$. As the first approximation, flow rate $Q^{(1)}=Q_{\max } / 2$ is selected with the corresponding calculations conducted of task 1 : $p(0)=p_{0}, T(0)=T_{0}, Q=Q^{(1)} . \quad$ The calculation determines the meaning of $p^{(1)}(L)$ of the pressure at the end of the pipeline $x=L$. There are two options possible:
if it turns out that $p^{(1)}(L)<p_{L}$, it means that the mass flow of gas $Q^{(1)}$ in the first approximation was set too low, and should be increased; and as the second approximation $Q^{(2)}=\left(Q_{\max }+Q^{(1)}\right) / 2$ should be applied;
if it turns out that $p^{(1)}(L)<p_{L}$, it means that the mass flow of gas $Q^{(1)}$ in a first approximation has been selected too large, however, it should be reduced and $Q^{(2)}=\left(0+Q^{(1)}\right) / 2$ should be taken as the second approximation. After the selection of the mass flow of the second approximation, the calculation by the solution method of the task 1 is repeated again, and the new value $p^{(2)}(L)$ at the end of the gas pipeline is determined.

The algorithm, described above, where choice $Q^{(\mathrm{j})}$ is determined by dividing the iterations of an interval in half, converges quickly and allows to find a value $Q_{*}$ for which the pressure $p_{*}(L)$ at the end of the gas pipeline will be very close to the pressure $p_{L}$ presented in the task, i.e. different from it but no more than in predetermined error value.

## 4. Example of Numerical Calculation: the "South Stream" Gas Pipeline

As an example, we present the results of the calculation of thermal-hydraulic conditions of work of one of the embodiment of a deep pipeline "South Stream", which is designed to ensure the supply of gas to European countries via the Black Sea and the Balkans (Fig. 1).

The peculiarity of this pipeline is that about 960 km of pipe is on the bottom of the sea, of which 600 km - at a depth of $\approx 2000 \mathrm{~m}$. In addition, the pipeline is characterized by high and and ultrahighpressure (the pressure at the beginning of the pipeline is 31.5 MPa , and at the end of the pipeline is $\approx 7.5 \mathrm{MPa}$ ).

The diameter of the pipeline is $32-33^{\prime \prime}$ (inner diameter is $735-758 \mathrm{~mm}$, and the wall thickness is $\approx 40 \mathrm{~mm}$ ), the design capacity of the 4 -trunk pipeline is equal to 63 billion $\mathrm{m}^{3}$ per year.


Figure 1 - Profile of the Underwater Section of the Pipeline "South Stream" from the compressor station "Coast" to "Varna" compressor station

Fig. 2 shows the distribution of gas pressure $p(x)$ and temperature $T(x)$, resulting from the calculation by the method described above.



Figure 2 - The pressure distribution $p(x)$ (top), temperature $T(x)$ (bottom) along the length of the underwater section of the pipeline

The figure shows that the pressure in the pipeline on the land area ( $0 \leq x \leq 30 \mathrm{~km}$ ) due to frictional forces is reduced, and then the descent below sea level ( $30<x \leq 60 \mathrm{~km}$ ) from zero to the depth of 2000 m increases almost in 1.5 MPa . Such increase is unusual for flat gas pipelines, in this case it is due to the profile of pipeline, i.e. caused by the weight force of compressed gas. If the gas density is low, then as a rule, the plain gas pipelines are characterized by equation:

$$
z_{1}-z_{2} \ll \int_{x_{1}}^{x_{2}} \frac{d p}{\rho g}
$$

i.e. the geometric difference of pressures is negligible compared with the difference between the piezometric
head, so the Bernoulli equation, is written for compressible medium,

$$
\left(\frac{\alpha_{K} v^{2}}{2 g}+\int \frac{d p}{\rho g}+z\right)_{1}-\left(\frac{\alpha_{k} v^{2}}{2 g}+\int \frac{d p}{\rho g}+z\right)_{2}=\int_{x_{1}}^{x_{2}} i d x
$$

where $i$ is the hydraulic gradient, the elevations can be neglected. However, it is not so in the case. If you do not take into account the change of velocity head and the losses of friction head, then the value should be maintained.

$$
\int \frac{d p}{\rho g}+z=i d e m
$$

Since the slope of the gas pipeline in the area under the water $z(x)$ decreases, the pressure $p(x)$ increases. It can be seen from the graph of the pressure distribution between $500<x \leq 650 \mathrm{~km}$ of the pipeline.

On a relatively horizontal bottom section of $60<x \leq 500 \mathrm{~km}$, the pressure in the gas pipeline monotonically decreases due to the friction forces. On a plot of lifting of $500<x<650 \mathrm{~km}$, the pressure decreases even faster, as elevation $z(x)$ increases.

The distribution of the gas temperature $T(x)$ has the following characteristics:
for the first 150 km , where the pipeline is lowered to the sea surface, the gas due to the strong heat exchange with the environment is quenched to a temperature close to the temperature of the water (at the bottom of the Black Sea, the water temperature during the whole year is around $+9^{\circ} \mathrm{C}$ );
on the flat bottom, the cooling gas becomes less intense because of the heat exchange with the environment $K_{T} \approx 20 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$, gas continues to cool;
on the upstream portion ( $500-650 \mathrm{~km}$ ), the gas temperature decreases sharply to 279 K and becomes lower than the temperature of the sea water due to the gravity forces;
at the last shallow waters plot ( $650-940 \mathrm{~km}$ ) the gas transported is heated a bit due to the heat exchange with the environment, so the temperature at the end of the plot is $\approx 280 \mathrm{~K}\left(\approx+7^{\circ} \mathrm{C}\right)$.

## 5. Condition for the Stationary Solutions Existence

Let us consider the case when the main determinant $\Delta$ of the system of equations (9) is equal to 0 , i.e. $\Delta=a_{1} b_{2}-a_{2} b_{1}=0$. In this case, the solution either does not exist, or is not the only one.

Equating the determinant $\Delta$ to zero, we obtain the equation for determining the critical mass flow rate $Q_{c r}$, or the critical velocity $v_{c r}$. We have:

$$
\begin{aligned}
\Delta=C_{p} & \left\{\frac{1}{\left(\rho v_{c r}\right)^{2}}-\frac{R T}{p^{2}}\left[Z-p\left(\frac{\partial Z}{\partial p}\right)_{T}\right]\right\}+ \\
& +\frac{R^{2} T}{p^{2}}\left[Z+T\left(\frac{\partial Z}{\partial T}\right)_{p}\right]^{2}=0,
\end{aligned}
$$

where we find the value of the critical velocity:

$$
\begin{equation*}
v_{c r}=\sqrt{\frac{C_{p}}{C_{V}} \frac{Z R T}{1-\frac{p}{Z}\left(\frac{\partial Z}{\partial p}\right)_{T}}} . \tag{11}
\end{equation*}
$$

Expression (11) is a so-called adiabatic speed of sound in real gas. In a perfect gas $Z \equiv 1$, so $(\partial Z / \partial p)_{T} \equiv 0, \quad C_{p}=C_{V}+R, \quad C_{p} / C_{V}=\gamma \quad$ is the adiabatic index (for methane is $\gamma \approx 1,31$ ) and the velocity of the sound is determined more simply and depends only on temperature: $\left(v_{c r}\right)_{p e r f}=\sqrt{\gamma R T}$. Let us suppose, for example, $\quad C_{V}=1900 \mathrm{~J} /(\mathrm{kg} \cdot \mathrm{K})$, $C_{p} \approx 2450 \mathrm{~J} /(\mathrm{kg} \cdot \mathrm{K}) ; \quad R=500 \mathrm{~J} /(\mathrm{kg} \cdot \mathrm{K}), \quad$ then $\left(v_{c r}\right)_{\text {perf }} \cong 435 \mathrm{~m} / \mathrm{s}$.

Using the equation of the state $p=Z(p, T) \rho R T$ of real gas, the formula (11) can calculate the speed of sound in a real gas. Table 3 shows the speeds of sound in the gas, $R=500 \mathrm{~J} /(\mathrm{kg} \cdot \mathrm{K})$ at a temperature $T=293 \mathrm{~K}$ for different pressures.

Table 3 - The Dependence of Sound velocity from the Pressure ( $\boldsymbol{T}=\mathbf{2 9 3} \mathrm{K}$ )

| $p_{c r}, \mathrm{MPa}$ | 0,1 | 5,0 | 10,0 | 15,0 |
| :---: | :---: | :---: | :---: | :---: |
| $v_{c r}, \mathrm{~m} / \mathrm{s}$ | 435 | 384 | 435 | 470 |
| $p_{c r}, \mathrm{MPa}$ | 20,0 | 25,0 | 30,0 | 35,0 |
| $v_{c r}, \mathrm{~m} / \mathrm{s}$ | 525 | 570 | 630 | 700 |

With increase of the pressure $p_{c r}$, at first it decreases, passes through a minimum, then at large pressures is steadily increasing and may exceed the value of $700 \mathrm{~m} / \mathrm{s}$.

Thus, $\Delta=0$ if the gas flow velocity in the pipeline reaches the local speed of sound. Typically, the velocity of the gas in the pipeline is $5-15 \mathrm{~m} / \mathrm{s}$, which is significantly less than the critical velocity $v_{c r}$. It can be possible only if the pipeline ruptures or discharges of gas fall into the atmosphere through the so-called candle speeds may reach the local speed of sound. In critical sections $d p / d x \rightarrow \infty$, so they develop the discontinuity pressure, or as they are called, shocks, generating shock waves.

## Conclusions

1. Calculation of steady-state operation of the gas pipeline with an arbitrary profile is reduced to solving a system of two coupled ordinary differential equations explicitly solved for the first derivatives of the unknown functions - pressure and temperature that are dependent on coordinate along the axis of the pipeline. The right sides of these equations can be expressed as derivatives of the compressibility of gas pressure, and temperature can be explicitly represented by the equation of the state of natural gas (including high and ultra-high pressures). Information on the coefficients of heat transfer and hydraulic resistance, as well as the external temperature and environmental properties is also required.
2. The problem of calculating the distribution of pressure and temperature at their given initial values and the known mass flow rate of the gas is reduced to the solution of the initial Cauchy problem for this system of differential equations. The problem of the calculation of these distributions from the known pressures at the beginning and end of the section of the pipeline is reduced to the solution of the boundary value problem for the same system of equations, which can be found by the method of iterations, as it is shown in the paper.
3. The solution to both problems exist, if in the case of each section of the pipeline, the subsonic flow regime is realized. Explicit formulas for the speed of sound in a real gas are presented. It is shown that high pressures can exceed the speed of sound in 1.2-1.7 times of the speed of sound in a proper case, moderate pressures, including those calculated under the standard conditions.

The author thanks to prof. M.D. Serediuk for scientific discussions and constant attention.

## References

[1] Charnyj, IA, 1975, Gazovaja dinamika, Nedra, Moscow.
[2] Jufin, VA (ed.) 1978, Truboprovodnyj transport nefti i gaza, Nedra, Moscow.
[3] Lurie, MV 2012, Matematicheskoe modelirovanie processov truboprovodnogo transporta nefti, nefteproduktov $i$ gaza, Publishing Center Gubkin Russian State University of Oil and Gas, Moscow.
[4] Novoselov, VF, Gol'janov, AI \& Muftahov, EM 1982, Tipovye raschety pri proektirovanii i jekspluatacii gazoprovodov, Nedra, Moscow.
[5] Kirillin, AV, Sychev, VV \& Shejndlin, AE 1983, Tehnicheskaja termodinamika, Jenergoatomizdat, Moscow.
[6] Genick, Bar-Meir 2007, Fundamentals of Compressible Fluid Mechanics, Minneapolis, USA.
[7] Reid, RC, Prausnitz, JM \& Poling, BE 1987, The Properties of Gases and Liquids. 4th edt, McGraw-Hill.
[8] Batchelor, GK 2002, An introduction to fluid dynamics, 12th edt, Cambridge University Press, Cambridge.

# Розрахунок профілів газопроводів з високим або надвисоким тиском у стаціонарному режимі 

М.В. Лур'є<br>Російський державний університет нафти і газу імені І.М. Губкіна;<br>Ленінський проспект, 65, Москва, 119991, Російська Федераиія


#### Abstract

Розглядаються газопроводи, умови роботи яких значно відрізняються від умов більшість існуючих систем. Бралися до уваги газопроводи з великою різницею у висоті, що працюють за високих (5-15 МПа) і надвисоких (до $25-35$ МПа) тисків. Перш за все, розглядаються глибинні газопроводи, що долають високі перевали. Наведено теорію i алгоритм розрахунку стаціонарних режимів таких газопроводів.


[^1]
[^0]:    * Corresponding author:
    lurie254@gubkin.ru
    mikhail.lurie@mail.ru
    © 2013, Ivano-Frankivsk National Technical University of Oil and Gas. All rights reserved.

[^1]:    Ключові слова: високий і надвисокий тиск, газопровід "Південний потік", диферениіальне рівняння, коефічієнт Джоуля-Томсона, профіль газопроводу, рівняння стану, чисельний алгоритм.

